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# MEASUREMENTS AND ADVENTURES AT THE INTERFACE OF MATHEMATICS AND ENVIRONMENTAL EDUCATION[1]

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*Abstract:* STEAM subjects are becoming increasingly important in 21st-century education as they provide opportunities to develop problem-solving skills. Through experiential learning, students can learn science through play. Learning by doing at an early school age also makes math and science lessons fun. The current Hungarian National Curriculum (2020) strongly emphasizes the development of key competencies, but science competencies are not part of the current curriculum. Thus, teaching environmental studies in grades 1–2 has been discontinued, and students' scientific competence in grades 3–4 must be developed in other subjects. Mathematics and environmental education are linked in several areas, one of which is measurement, including the concepts of perimeter and area. In teaching the concept of perimeter, we aim to give children numerous ways of experiencing that perimeter is the length of a line that bounds a shape in the plane. When developing the concept of area, the first step in defining the area of a rectangle is always to start from the actual coverage, and then the measurement of the area of the rectangle can be the basis for further area definitions. These concepts should be explored not only on the plane but also on a finite spherical surface (e.g. a Lénárt sphere) through playful activities that help students navigate and measure the globe.

*Keywords:* STEAM-based problem-solving, measurement, mathematics, environmental education

## INTRODUCTION

21st-century skills are essential in all areas of life, whether at work or in our personal lives. That is why it is so important for students to leave school with

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the right competencies. The development of competencies is not only a task for secondary school, as many people believe, since most of them are already present in primary school.

The key competencies are also included in the Hungarian National Curriculum, in line with the European Union's requirements. However, the key competencies of science are missing from the new basic curriculum, and as a result, the number of lessons in subjects that support the foundation and development of scientific thinking has been reduced. In this study, we will address the situation of the subject of environmental studies at the primary level, which has been discontinued in grades 1 and 2. This change has led to the integration of environmental content into other subjects, including mathematics. The only positive aspect of this situation is that it allows STEAM-based problem-solving with environmental content in mathematics lessons.

In this study we have collected good practices from Hungary that can help practicing teachers in teaching measurement, focusing on the environmental content and the importance of learning and understanding mathematics.

## LITERATURE REVIEW

### *Competencies for the 21st Century*

The terms transversal competencies, soft skills, and key competencies are often used instead of 21$^{st}$-century competencies. Not only do we read about different terms in different literature or documents of international organizations, but also different definitions. Accordingly, there are different frameworks, which largely overlap, but with some minor differences. In the grouping of competencies and areas of competence into structural categories, such as cognitive, interpersonal, intrapersonal, and global citizenship, differences can be observed depending on the mission of the organizations developing each framework and their objectives in using the framework.

According to M. Binkley et al. (2012):

> Twenty-first-century skills are abilities and attributes that can be taught or learned in order to enhance ways of thinking, learning, working and living in the world. The skills include creativity and innovation, critical thinking / problem-solving / decision-making, learning to learn / metacognition, communication, collaboration (teamwork), information literacy, ICT literacy, citizenship (local and global), life and career skills, and personal and social responsibility (including cultural awareness and competence).

Based on Suto and Eccles (2014), the "inter-disciplinary skills most commonly regarded as essential for the 21st Century are problem-solving, ICT operations and concepts, communication, collaboration, and information literacy."

As stated by the OECD when it established the PISA framework, the "competency is more than just knowledge and skills. It involves the ability to meet complex demands, by drawing on and mobilizing psychosocial resources (including skills and attitudes) in a particular context" (OECD, 2005).

According to UNESCO (2016), transversal competencies can be grouped into six areas of competence: interpersonal skills, intrapersonal skills, media and information literacy, critical and innovative thinking, global citizenship, and other skills (*Figure 1*). Each area has several competencies.

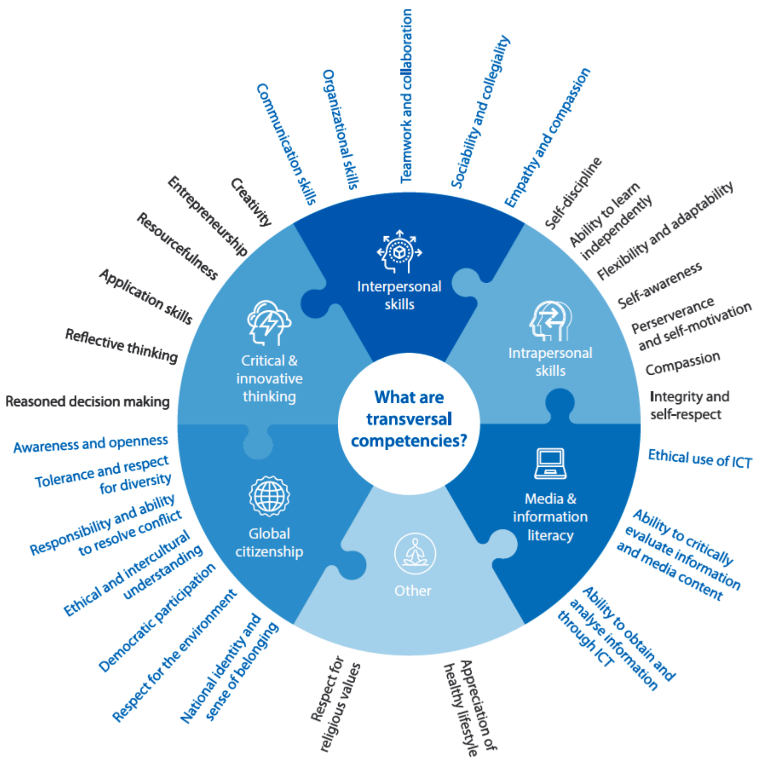


*Figure 1: Transversal competences (UNESCO, 2016)*

As defined by the European Commission (2019) "key competencies are a combination of knowledge, skills, and attitudes. The key competencies are developed throughout life, through formal, non-formal and informal learning in different environments, including family, school, workplace, neighborhood, and other communities."

Whichever framework we look at, they all have one thing in common: they all seek to respond to the challenges of the future of the world. Taking into account the current problems – including the incredibly rapid technological development, which is becoming even more important with the use of artificial intelligence, the effects and challenges of globalization, demographic changes, environmental sustainability, the uncertain political situation – the aim is to develop competencies that can be acquired to mitigate and solve them.

Problem-solving is the ability to identify a problem, determine its cause, and come up with all possible solutions to solve it. Problem-solving, along with other transversal competences, should be acquired at a young age (Bağçeci & Kinay, 2013). In preparing students for the future, relevant and innovative problems that are challenging, yet solvable and manageable, should be built into the learning process, thus fostering the development of problem-solving skills (McKenna, 2014). Developing problem-solving skills is, by its very nature, a core task of STEAM subjects, of which mathematics is considered the foundation of other sciences yet is often overlooked in STEAM activities (Maass et al., 2019; Mayes, 2019; Roberts et al, 2022). However, this may have negative consequences if students do not see meaningful connections between learning mathematics and other STEAM subjects, as they may then lose interest not only in mathematics but also in other subjects (Kelley & Knowles, 2016). In the real world, STEAM-based problems are increasingly covering broader areas than just disciplinary content and practice, meaning that non-STEAM dimensions of the problems need to be addressed and prepared for.

### *Key Competencies and the Hungarian National Curriculum (NAT)*

The European Parliament and Council Recommendation on Key Competencies for Lifelong Learning was adopted in 2006. As a result of this document, the key competencies – combinations of knowledge, skills and attitudes – were introduced in the National Curriculum of Hungary in 2007, and special emphasis was placed on competence-based teaching and learning. The development of key competencies has been an ongoing process in public education since then, as confirmed by the subsequent National Curricula (*Table 1*).

In *Table 1*, it can be observed that the 2020 National Curriculum lacks scientific and technical competencies, which are the knowledge and skills that describe processes in the environment and predict their expected outcomes. This poses a problem for the teaching of science subjects, as the change has resulted in a reduction in the number of hours taught in these subjects.

*Table 1: Key competencies (NAT 2007; NAT 2012; NAT 2020)*

| NAT 2007 | NAT 2012 | NAT 2020 |
|---|---|---|
| Native language communication | Native language communication | Communication competencies (native and foreign language) |
| Foreign language communication | Foreign language communication | |
| Mathematical competences | Mathematical competences | Mathematical, thinking competencies |
| Nature scientific competence | Nature, scientific and technical competence | - |
| Digital competence | Digital competence | Digital competences |
| Social and civic competence | Social and civic competence | Personal and interpersonal relationship competencies |
| Initiative and entrepreneurial competence | Initiative and entrepreneurial competence | Employability, innovation, and entrepreneurship competences |
| Aesthetic-artistic awareness and expression | Aesthetic-artistic awareness and expression | Competences for creativity, creative work, self-expression and cultural awareness |
| Effective, independent learning | Effective, independent learning | Competences of learning |

In Hungarian primary schools, environmental studies is one of the natural science subjects in the lower grades, which builds on students' curiosity to learn about phenomena and processes in the wider environment. Environmental studies provide the basis for a scientific way of thinking. The already low number of lessons in the subject has been further reduced following the introduction of the National Curriculum 2020, so there are no environmental lessons in grades 1 and 2 (*Table 2*), and the previous environmental content is only integrated into other subjects such as Hungarian language and literature, mathematics, ethics/faith and morals, technology and design, visual culture (Nat, 2020). The positive side of this change is that STEAM activities can integrate mathematics into the environmental content by working on real problems that are age-appropriate for children.

*Table 2: Proposed number of lessons for mathematics and environmental studies based on NAT 2012 and NAT 2020*

| | Grade 1 | Grade 2 | Grade 3 | Grade 4 |
|---|---|---|---|---|
| Mathematics NAT 2012 | 4 | 4 | 4 | 4 |
| Mathematics NAT 2020 | 4 | 4 | 4 | 4 |
| Environmental Education NAT 2012 | 1 | 1 | 1 | 1 |
| Environmental Education NAT 2020 | | | 1 | 1 |

In the 2020 National Curriculum for Mathematics for grades 1–2, several topics combine mathematical and environmental content:

- Sorts, creating and investigating sets,
- System design, system building,
- Assertions,
- *Use of measuring instruments, methods of measurement,*
- Spatial and Plane Orientation,
- Recognising relationships, connections, regularities, and
- Observing data.

Based on the development tasks and the knowledge listed, the topic Use of measuring instruments and methods of measurement is one of the most suitable topics to teach students about measurement, the use of measuring instruments and measurement methods and to implement STEAM-based problem-solving.

### *Teaching Measurement in Primary School*

Measurement also plays a variety of roles within mathematics in lower school work. In other subjects (e.g. environmental studies, technology, drawing), measurement is used primarily as a method and tool for learning. In mathematics, it is not only used as a tool but also to build two important concepts. Measurement is the empirical basis of the number concept, and operations are also linked to this number concept. The development of quantity concepts also relies on practical measurements. It is not enough to name a quantity; the content of the concept is closely linked to the measuring instrument and the measurement procedure. It is also part of the work at the primary level to develop elementary skills in practical measurement and to develop an awareness of standard units.

We follow the usual steps in the development of geometric concepts of quantity (e.g. length, perimeter, area):

1. sensory experience of quantity, comparisons by size through direct experience: grasping and expressing the difference;
2. measuring quantities together; learning the tools and procedures to be used for this purpose; thus, detecting small differences, grasping and expressing equality; deciding how much larger or smaller is a question of how much larger or smaller; and
3. measuring by units; (measuring by multiples of a unit); increasing the accuracy of measurement by measuring by fractions of a unit, (lower and upper approximation). (C. Neményi, 2007)

Working out the concept of length means, first of all, experiencing and then linking the different quantities of length: height, length, width, thickness, depth, and circumference (including perimeter) separately in school activities. By school age, children have already had sensory experiences of each of these contents and have also had the opportunity to experience direct or indirect measurement.

We emphasize that the perimeter is not an independent quantity, but a length: the length of the line bounding the plane shapes. This length can be measured just like any other length dimension. At most, it is sometimes more convenient to measure the sides separately and add the resulting measurements; at other times, if there are equal sides, the addition can be further simplified by substituting multiplication. We do not learn a "formula" for "calculating" the circumference of a rectangle or square, for example, not only because it would not be the correct way of learning at the right age. Nor is it because it would obscure the point: the concept of perimeter (C. Neményi, 2007).

The concept of area is more difficult than that of length. Young children can have a wide range of sensory experiences of what it means to walk a very long way, to climb a high couch, to jump down a high staircase, or to hug a thick tree. But they have less experience of the volume of the area. As with length, it's useful to gain experience of the area through some kind of movement. *Do not tie the concept of area to a shape!* In real measurements, it is important to make children aware that the unit of measurement does not depend on the shape of the unit, but only on its size, and that they should therefore measure units of the same size but different shapes. When determining the area of a rectangle, start by working out the simplifying procedure, always starting from the actual puzzle first. Measuring the area of a rectangle can be the basis for further area definitions (C. Neményi, 2007).

It is also worth introducing the Lénárt sphere to children. Research has shown (Gambini, 2021) that if children are introduced to plane geometry and spherical geometry at the same time, it becomes natural for them to accept and compare the concepts of plane geometry and spherical geometry. Spherical geometry is a mathematical and pedagogical counterpart to plane geometry. In fact, it surpasses plane geometry in one very important respect: it is not based on infinite but on finite surfaces. Moreover, the understanding of the concepts involved is greatly facilitated by the fact that pupils can experiment on a real sphere (an orange, a paper ball or a Lénárt sphere). At the same time, the comparison of the plane and the sphere also encourages pupils to ask questions of each other and their teachers, to explore and experiment independently, and to compare their ideas with those of others. Another advantage of spherical geometry is that many of its basic concepts are also covered in another subject, geography. Learning spherical geometry can contribute to an easier orientation on the globe and a better understanding of basic geographic concepts (Lénárt, 2009).

## GOOD PRACTICES FROM HUNGARIAN EDUCATION

### *Game to Experience the Concept of Length*

The online logic puzzle game Shingoki (Semaphores) is also a great way to experience the concept of length, gain sensory experience, and use direct comparison. The rules are very simple. You have to draw lines between the dots to form a single loop without crossings or branches. The loop should pass through all black and white circles in such a way that:

- White circles must be passed through in a straight line;
- Black circles must be turned upon; and
- The numbers in the circles show the sum of the lengths of the 2 straight lines going out of that circle.

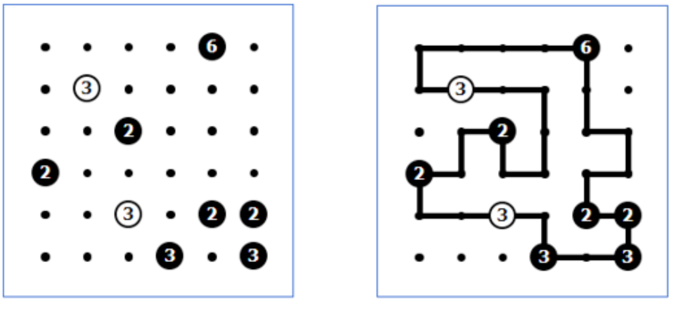

*Figure 2: A Shingoki puzzle and its solution (Source:https://www.puzzle-shingoki.com/)*

### *Games to experience the concept of area*

The online logic game Shikaku (also known as Rectangles) is a great way to determine the area of a rectangle, to try out the actual puzzle, and to experience it. The rules are simple. You have to divide the grid into rectangular and square pieces such that each piece contains exactly one number, and that number represents the area of the rectangle.

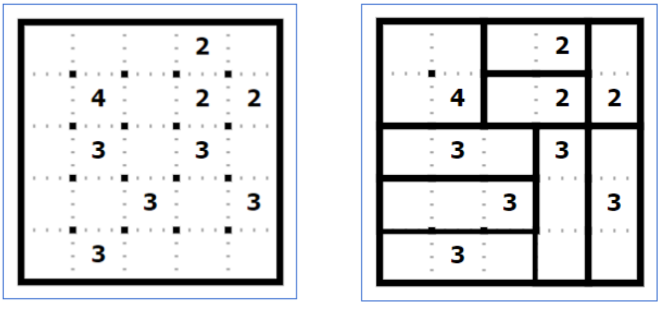


*Figure 3: A Shikaku puzzle and its solution (Source: https://www.puzzle-shikaku.com/)*

For trying out different polygon puzzles and covering a given area, the Geogebra-based online game Playing with Pentominoes (*Figure 4*) is a great way to do it. In the game, two players have to place 12 pentominoes on an 8 by 8 grid on the game board. The players can place one element at a time, and the elements can be rotated and mirrored, but the elements must not hang off the track and must not overlap. The game continues until the track is full or the set of elements is exhausted. The last person to place a pentomino on the playing field wins.

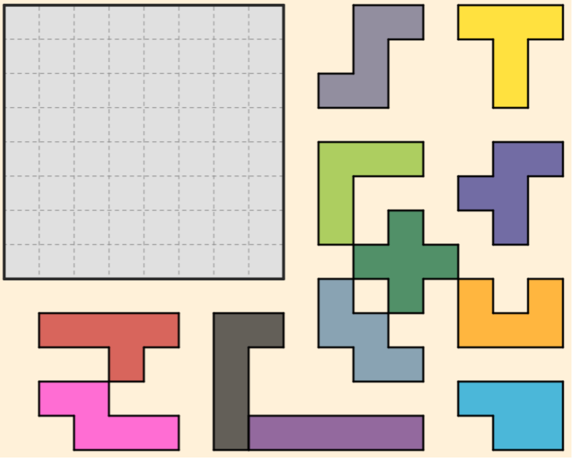

*Figure 4: Playing with Pentominoes (Source: https://tananyag.mdoe.hu/mod/book/view.php?id=37&chapterid=594)*

The GeoGebra-based Mosaic Puzzle game, of which there are several versions, helps to build the concept of area. There is one in which you can cover the plane with diamonds only, one in which you can cover the plane with two types of plane (diamond and regular triangle), one in which there is a "blank" area to cover, and one in which pre-drawn lines help you to cover the area. In the case of the exercise in *Figure 5*, the area of an empty hexagon is to be covered by congruent triangles and congruent quadrilaterals (diamonds). In the game, the plane objects can be dragged into position and rotated to the correct position. The hexagon can be covered by the 6 blue diamonds and 12 orange triangles given.

The students will experience the gap-free and overlap-free covering of a plane. Since the area of a diamond is equal to the area of two triangles, they can also think about how many pieces of the same shape would be needed if only diamonds or only triangles were used for the puzzle. They can also make the important deduction that triangles with half the area would need twice as much to complete a hexagon as diamonds. This proportional reasoning will be of great help to children later on when learning to convert units of measurement.

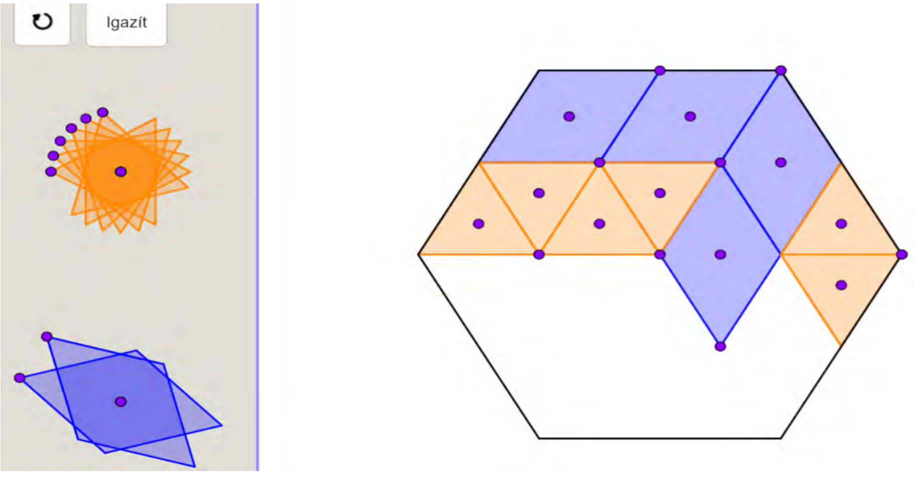


*Figure 5: Mosaic puzzle (source: https://tananyag.mdoe.hu/mod/book/view.php?id=39&chapterid=562)*

### *Playful Activities on the Sphere*

To illustrate the sphere for primary school children, choose orange because it fits easily in a child's hand and can be drawn on with a marker pen. To experience the concept of length on the sphere, the children should first be asked to place a dot on the orange, this dot is called the North Pole. Tell the children that there is a penguin who wants to be the furthest away from the North Pole. Ask the children to place on the orange where the penguin might be located. Let's call this point the South Pole! Next, let's tell the children about another character, a turtle who hates the cold and wants to be as far away as possible from both the North and South Poles. Ask the children to draw on the orange where this turtle might be located (*Figure 6*). After completing this task, it is also worth discussing with children how many places they can draw the turtle on the orange.

*Figure 6: Where does the turtle live?*

Then ask the children to take the oranges in their hands and try to measure around them (*Figure 7*). After measuring, it is worthwhile to experience and discuss with the children that the distance on the sphere (orange) cannot be greater than the distance they measured on it.

*Figure 7: Measurement of orange*

The activities presented are an excellent way for children to experience the very important difference between the plane and the sphere, namely that the sphere, and therefore spherical geometry, is not infinite, unlike the plane, but is based on a finite surface.

## RESULTS

In our research, we wanted to demonstrate that children's skills and knowledge levels can be improved using different geometric play activities and the good practices described above. Experience has also shown that when teaching geometry, it is worth starting with games about bodies and real objects, and by the principle of perceptual variety, it is worth showing children as many different shapes as possible of the geometric structure to be taught (C. Neményi, 2007). Through games, children can experience the connections within a given system and later, based on experience, they can also visualize the clarification of this structure (Dienes, 1999). Games and playful activities lead to the experience of geometric properties and geometric relations and relationships, as well as to the learning of geometric shapes. On this basis, we used different geometric games and activities during 6 lessons (Kéner, 2023). The class that participated in the research was a lower 3rd-grade class at a practical school in Budapest. The participating students are not representative of the population, so the results are not generalizable, but they do provide an opportunity to make observations that can be verified by increasing the sample size.

To test the effectiveness of the teaching-development process, a test was set up. The eDia online diagnostic assessment system developed by the Educational Theory Research Group of the University of Szeged was chosen as the test platform (Molnár & Csapó, 2019; Molnár, et al., 2021). Students were able to access the series of tasks for the test on any device (desktop, laptop, tablet, or smartphone). For the tasks for grades 1 – 3, the instructions are not only readable but also listenable, so students' reading performance does not affect their math performance. For the substantive part of the test, 8 tasks were selected, containing a total of 31 items. 19 plane geometry items and 12 solid geometry items were included in the test.

We administered this test to 25 third-grade students in a school in Budapest, Hungary, to see if the test could be used to measure children's plane geometry and spatial geometry skills. The reliability of the test was good (Cronbach's $\alpha$=0.85), so we used this test as a first step in the implementation of our development program. Thus, students in the class from which the children participated in the program took a pre-test as a first step. The purpose of the pre-test was not only to assess the children's current level of skills and knowledge but also to select the students in the control group and the experimental group.

Based on the pre-test, pairs of students were formed in the research class, i.e. students with the same or nearly the same score were paired, one in the experimental group and one in the control group. Occasionally, two children were assigned to the control group in addition to one child, so that there would be enough comparable performance in case of dropouts or absenteeism. At the end of development, all the children took the post-test, but only the data of the originally selected children were included in the analysis. Thus, 7 young children from the experimental group and 9 young children from the control group.

To measure the changes in the performance of the children in the study, we included two spatial geometry and three plane geometry tasks from the pre-test in the post-test. These included both easier and harder items. The other tasks, whose results did not allow us to detect a possible developmental effect, were replaced by more challenging tasks. Which items were harder, and which were easier in the pre-test was determined based on the children's performance on each task by averaging the scores on each item. Thus, the post-test consisted of 9 tasks, with a total of 32 items. 15 items were around plane geometry, while 17 items were around solid geometry.

To examine progress, only the twenty items included in both tests were analyzed. There was no measurable difference between the performance of the two groups on the pre-test, so it can be said that the children were in the same place in terms of their knowledge and ability levels in both groups. However, the results of the post-test show that the experimental group performed significantly better than the control group because of the development (*Table 3*).

Although the performance in the post-test was also examined for the full test (*Table 3*), this result is not comparable with other results, as it does not only include the same items as in the pre-test. However, this test also included more challenging items than the pre-test. When comparing the results of the two groups on the full post-test, it can be seen that the experimental group maintains a significantly better performance than the control group on the full test. The results show that the experimental group achieved significantly better results than the control group because of the program we designed and implemented.

*Table 3: Comparison of the performance of the experimental and control groups on each test*

| Test | Group | Headcount (N) | Average (percent.+point) | Scatter | Significance level (p) |
|---|---|---|---|---|---|
| Pre-test | Experimental | 7 | 77.00 | 5.78 | 0.201 |
| | Control | 9 | 79.40 | 5.05 | |
| Items common to the pre-test and post-test | Experimental | 7 | 70,71 | 9.32 | 0.216 |
| | Control | 9 | 74.44 | 8.82 | |
| Items common to the post-test and pre-test | Experimental | 7 | 87.14 | 9.51 | 0.008* |
| | Control | 9 | 73.89 | 9.61 | |
| Post-test | Experimental | 7 | 82.49 | 9.83 | 0.028* |
| | Control | 9 | 71.33 | 11.57 | |

$p<0.05$

If we look at the results not by comparing the two groups, but by comparing the pre-test and post-test scores of both the experimental and control groups, we see that the control group did not improve relative to itself, while the experimental group performed significantly better relative to itself on the post-test than on the pre-test (*Table 4*).

*Table 4: Change in each group's performance relative to itself between the two surveys*

| Group | Test | Headcount (N) | Average (percentage point) | Scatter | Significance level (p) |
|---|---|---|---|---|---|
| Experimental group | Items common to the pre-test and post-test | 7 | 70.71 | 9.32 | <0.001* |
| | Items common to the post-test and pre-test | 7 | 87.14 | 9.51 | |
| Control group | Items common to the pre-test and post-test | 9 | 74.44 | 8.82 | 0.455 |
| | Items common to the post-test and pre-test | 9 | 73.88 | 9.61 | |

p<0.05

The results of the study show that the 3rd-grade students who were involved in activities involving geometric play and good practice during the study period improved their knowledge and skills. Considering the results obtained, we intend to present further good practices in geometry to practicing teachers in the framework of professional days, and to put more emphasis on playful activities and their practical usefulness for our teaching students by restructuring the geometry pedagogy course structure in our department.

## CONCLUSION

The development of 21st-century competencies is now an important task for teachers, even in primary schools. These competencies are not subject-specific and are becoming indispensable for students in all walks of life. This paper focuses on problem-solving competencies and how they can be developed.

The Hungarian National Curriculum, which will be in force from 2020, will no longer provide the opportunity to teach the former environmental studies subject in the 1st and 2nd grades of primary school due to the reduction in the number of lessons. Therefore, environmental content will have to be taught to pupils in mathematics lessons, among others. The positive side of this situation is that by using STEAM-based problem-solving in teaching lower secondary school pupils, environmental and mathematical content can be presented together, thus increasing the importance of mathematics – a necessary subject, as it is the basis of all science subjects – in the eyes of the pupils. In this study, we have focused on measurement, which is an integral part of both subjects. To develop problem-solving competencies and take into account the situation in Hungary regarding the teaching of environmental content, in this study we have presented Euclidean and non-Euclidean geometric good practices that can help to teach the topic of measurement.

In our research, using STEAM-based problem solving, we found that the use of geometric play activities has a developmental impact on students' knowledge and skills. Using the results obtained, we would like to incorporate more hands-on activities in the geometry teaching course of our department, hoping that the colorful methodological repertoire of future lower secondary teachers will help the development of their students.

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