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ALGEBRAIC STRUCTURES BY CREATING MIND MAPS WITH STUDENTS GIFTED IN MATHEMATICS

Abstract: Working with mathematically gifted students is the subject of many studies. In the literature, one can find various examples of the positive impact of the use of mind maps on learning by understanding and connecting concepts in appropriate schemes, but the impact of creating mind maps on the achievements of students gifted in mathematics has not been sufficiently researched. Having that in mind and that algebraic structures represent a teaching topic in which it is necessary for students to have adequate theoretical knowledge about these structures and relations between them, this method was implemented in order for students to connect the proper concepts in a scheme. For that purpose, (two) mathematics classes of systematization are conducted for the teaching topic on Algebraic structures in order for students to create two mind maps each (one for algebraic structures with one and with two binary operations and another for homomorphism). The effects of this approach to the systematization classes were examined by analyzing the students' success achieved in two fifteen-minute tests (before and after the systematization classes) where they had to mark the correct statements (precisely formulated algebraic structures and homomorphisms). The results obtained by statistical analysis indicate that the students achieved statistically significantly better results in the test held after the systematization classes. In other words, the creation of mind maps by students gifted in mathematics had a positive effect on systematization of knowledge about Algebraic structures and on students' achievement in mathematics (specifically Linear Algebra and Analytical Geometry). This result implies that teachers who work with students gifted in mathematics should seriously consider organizing mathematics classes where students will systematize and deepen their theoretical knowledge by creating mind maps.

Keywords: mind maps, students gifted in mathematics, algebraic structures, teaching mathematics.

INTRODUCTION

Many studies and empirical research support the positive impact of using mind maps on learning focused on establishing connections between different and related concepts (Budd 2004; Farrand, Hussain, Hennessey 2002). Thus, positive results can be found in the literature of the use of mind maps created by teachers or students during the adoption of teaching content in mathematics (Brinkmann

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2003). At the same time, it should be emphasized that the age of the students varies in different researches, from the youngest students of school age to high school students and participants in higher education (Farrand, Hussain, Hennessy 2002; Kovačević, Segedinac 2007). However, in most research, mind maps are used in heterogeneous classes of students, when it comes to students' achievements in mathematics. On the other hand, there is also a large amount of research related to gifted students in mathematics. The emphasis in those researches is mainly on: how to recognize these students, in particular at a younger age (Bicknell 2009); on mathematical giftedness and mathematical creativity (Parish 2014); on the development of a mathematics curriculum for students gifted in mathematics (Zmood 2014); in the choice and method of solving mathematical problems (Leikin 2009), etc. Therefore, although mind maps, as well as work with gifted students, represent the topics of a significant number of empirical researches, the amount of research that connects these two topics is practically negligible. Indeed, it is very difficult to find research that discusses mind mapping by gifted students in mathematics and the impact of mind mapping by gifted students in mathematics on their achievement in mathematics.

The aim of this research is reflected in the analysis of the impact of the methodological approach, which involves creating mind maps on behalf of students gifted in mathematics in order to deepen and systematize their theoretical knowledge, which is necessary for successfully solving concrete problems on the teaching topic of Algebraic structures.

THEORETICAL BACKGROUND

MIND MAPS

Throughout history, mainly due to the low technical-technological level of development, people used two-dimensional representations of their ideas to try to find a solution to a concrete problem, to perform classifications according to different criteria of various phenomena, or simply to present their ideas in a hierarchical order according to some principles. For these purposes, people used graphic representations of knowledge – mind maps, from the earliest times to the present day (Rhodes 2013). Mind maps are, formally, special diagrams that can be used in situations involving the need for learning and thinking in any form (Kovačević, Segedinac 2007). Using mind maps improves our intellectual potential: memory, thinking, and understanding and noticing relationships and connections between terms and concepts (Farrand, Hussain, Hennessy 2002; Papić, Aleksić, Kuzmanović, Papić 2015). As Buzan points out, the mind map as a powerful graphic tool can be the key to releasing the potential of the brain, and it has four basic characteristics, (Buzan, Buzan 1995):

- The subject of attention is crystallized in a central image.
- The main themes of the subject radiate from the central image as branches.
- Branches comprise a key image or keyword printed on an associated line. Topics of lesser importance are also represented as branches attached to higher level branches.
- The branches form a connected nodal structure.

Alamsyah (Alamsyah 2009) explains that mind map should have the following elements:

1) The center of the mind map is the main idea or idea.
2) The main branch or basic order ideas (BOI), the first level branch that radiates directly from the center of the map. Thoughts.
3) Branches, which are emanations from the main branch, can be written in all directions.
4) Words, using only keywords.
5) Pictures, using pictures they like.
6) Colors, using attractive colors on the map.

For creating a mind map, it is important to use the keywords. Keywords are words that represent the "trigger impulse" for more relevant associative meanings. Using keywords reduces the number of words that are used on the map but on the other hand does not reduce the quantity of information associated with those keywords. Sometimes, it can be difficult to find the keyword, if it is trapped in a sentence. On the other hand, another important aspect is that when we choose it, our mind "digs" deeper in search for new meanings. Using different colors is very useful and stimulating when creating mind maps (Kovačević, Segedinac 2007).

Of course, over the long period of development of human civilization, mind maps have evolved and are used in various segments of modern life. According to the authors of the book *Mind Maps* (Buzan, Buzan 1995; Buzan 1976), the number of people who began to use brilliant thinking and mind mapping grew by an almost logarithmic progression. Throughout history, examples of numerous creative people and thinkers who have used mind maps can be found. Some famous intellectuals and people who primarily used graphic-visual representations in their intellectual work were Leonardo da Vinci, then Albert Einstein who used mind maps in unconventional ways to create unconventional ways of thinking (Rhodes 2013). The famous and previously cited Tony Buzan, a British author, believes that literate and well-educated individuals are limited because they are unable to use many of the conceptual tools for thinking, including mind maps (Rhodes 2013).

According to the studies of cognitive psychology (Morita, Asada, Naito 2016), human understanding of knowledge is a complex and changeable imaging process. Psychology says that the human brain remembers images much more strongly than words. Humans have left and right brain hemispheres, which are responsible for different brain activities. The left hemisphere is responsible for words, logic, numbers, order, linearity, analysis, and lists, while the right is responsible for rhythm, imagination, colors, daydreaming, gestalt, and dimensions (Stanković, Ranđić 2008). Common methods of memorizing information force the individual's brain to work linearly and interfere with the natural functioning of the brain. The brain works by principle of association and based on that can connect an idea or data with many other ideas and concepts (Anokhin 1973). Conventional teaching methods better support the work of the left half of the brain compared with the right half of the brain, but using mind maps stimulates the work of both halves of the brain. In this way, the logical structures relate to imagination on paper, which is the basis for a mind map. The left side of the brain is activated by keywords on the map, while adding images, colors, and three-dimensionality activates the right brain hemisphere – "the right creative brain" (Svantesson 1992).

In today's insistence on quality education, more attention is given to the cultivation and promotion of students' active learning ability and their thinking ability. Since school-based learning is comprised from a set of multiple situations that involve solving problems, organizing data, taking notes, writing, and presentations, mind maps are offered as a tool for all these activities (Brinkmann 2003). Mind maps are considered an excellent tool for accelerating learning, creativity, solving complex problems, and saving time. Just designing mind maps imitates the work of the brain symbolically and visually on paper. They represent connections between concepts, which contributes to building better connections in the brain itself and better recall of information. Mind maps, therefore, reflect the natural functioning of the brain, because they have a branched radial structure branching from a central term (Buzan, Buzan 1999). According to some researches, mind maps have a positive influence on the understanding of abstract concepts (Roth, Roychoudhury 1992). The observations that the individual creating the map play an important role in the placement, assimilation, organization, and retention of data (Ornstein 1986; Ornstein 1991). Mind mapping promotes divergent and creative thinking (White, Gunstone 1992). Connections between different parts of the map can be obtained by linking different parts of the map with arrows. This makes it easy to examine patterns of thought and similarities and connections between information in different parts of the map.

Mind maps can be an essential tool for teaching and learning. To carry out the steps of constructing a mind map, we must first understand the content of knowledge, proceed to identify the core content, and divide it into main ideas and identify sub-ideas of each main idea (Buzan, Buzan 1999). Many studies point to the effectiveness of the mind map technique (Budd 2004; Farrand, Hussain, Hennessey 2002). Efficiency of use of mind map techniques when improving factual knowledge from written information was studied by Farrand, Hussain, and Hennessey (Farrand, Hussain, Hennessey 2002). The attention of researchers was focused on mind maps as a learning aid. The remembered content was stable in both groups, but the participants from the group that used the mind mapping technique remembered the content better after a week. The authors pointed out that this method has an advantage over conventional methods of learning, and that students were enthusiastic about this method, which lead to more effective training for the implementation of the curriculum. In the research of Budd (Budd 2004), mind maps are presented as a tool for overcoming traditional blackboard and chalk learning styles. The work shows the possibility of using mind maps for the purpose of different learning styles and renewing energy during the semester. The exercise was organized so that students create within one subject mind maps on the given teaching topic. In groups of three, students were asked to think about what the first step in the formation of mind maps is. During the exercise, the instructor moved among the students and gave them feedback on the process of creating the map. This research supported the idea of active learning, and students with higher scores agreed on the positive impact of learning based on mind maps. Nowadays there are numerous software tools that enable the creation of mind maps, such as: Coggle, Freemind, Xmind, MindMeister, MindManager, LucidChart, Microsoft Visio, ClickUp, etc.

It is also possible to use mind maps in mathematics education. According to Brinkmann (Brinkmann 2003), mind maps can help in organizing information, they can be used as an aid in memorizing content and its repetition, and then in connecting new information with the students' existing knowledge. They allow students' cognitive structures to become visible, promote creativity, and ultimately help students to see the connection between mathematics and the real world. Although, as the author points out, mind maps are rarely used in mathematics education, feedback indicates that students who were not good at mathematics benefited from mind mapping. They understood the relationships and connections between mathematical concepts while creating a mind map (Brinkmann 2003). Mind maps made a strong impression on students who usually memorize. They turned such habits into meaningful learning (Arifah, Suyitno, Rachmani Dawi, Kelud Udara 2020).

STUDENTS GIFTED IN MATHEMATICS

The concept of giftedness is popularly considered as a concept that articulates the highest level of intelligence determined by IQ tests. Giftedness is a much wider concept, which refers to an alignment which is both cognitive and emotional and includes unique developmental aspects, as well as familial and social aspects (Tamir 2012, as cited in Zedan, Bitar 2017).

Educational literature related to the issues of high mathematical ability, mathematical talents, mathematical giftedness, and mathematical creativity contain a variety of descriptive reports and instructional guidelines, but there are much fewer research reports that could be found on the issues related to mathematical talents and mathematical giftedness. Schoenfeld (Schoenfeld 2000; Schoenfeld 2002) expressed the two main purposes of research in mathematics education which could be maintained for the research in the field of mathematical giftedness and creativity. Those purposes are:

- First (theoretical) is to understand the nature of mathematical giftedness and mathematical creativity from the perspectives of thinking, teaching, and learning;
- Second (applied) is to use such understanding in improving mathematics instruction in a way that helps realize mathematical giftedness and encourage mathematical creativity.

According to Leikin and her colleagues (Leikin 2009; Leikin 2014; Leikin, Paz-Baruch, Waisman, Lev 2017), the domain of mathematical giftedness implies a collection of certain mathematical abilities and personal qualities. Students who are gifted in mathematics are described as students with strong problem-solving abilities, metacognitive abilities, creative mathematical thinking, and high ability/performance in mathematical problem-solving. Characteristics of students that can indicate mathematical giftedness usually include: an extraordinary curiosity for numbers and mathematical information, a capability to understand and implement mathematical concepts quickly, a distinctively high ability to recognize patterns and abstract thinking, flexibility and creativity in strategies for problem solution, an ability to move mathematical concept to an unfamiliar situation, as well as perseverance in solving challenging problems (Stepanek 1999). Mathematically gifted individuals possess intellectual characteristics, such as curiosity, the ability to visualize models, fast thinking, and metaphorical thinking (Deary 2000; Silverman 1997). Krutetskii (Krutetskii 1976: 77) implies that gifted and talented mathematics students have (among other capacities) “the ability for rapid and broad generalization of mathematical relations and operations, and flexibility of mental processes”. The teachers observed the different pace of mathematics learning, an intuitive mathematical knowledge in problem-solving, their interest in mathematics, the sense of humor and ability to think in more abstract terms than peers of the same age, as well as more mental flexibility and a discourse based on logical thinking characterized students gifted in mathematics (Bicknell 2008). According to the students, other aspects that confirmed their mathematical giftedness include success in competitions, competence with basic mathematical facts, speed of computational skills, problem-solving abilities, and capacity to work on “special projects” or on more/different work (than their classmates) to complete independently (Bicknell 2008; Subotnik, Robinson, Callahan, Gubbins 2012).

Sriraman (Sriraman 2009) claims that mathematical creativity could be considered as the main mechanism of the growth of mathematics as science. Mathematical creativity is also mentioned as a characteristic among students gifted in mathematics, even though there is no commonly accepted definition of that term (Plucker, Beghetto, Dow 2004; Singer, Sheffield, Leikin 2017). Other studies take a different approach to creativity and adopt the concept of cognitive flexibility, which is explained as an interlude between cognitive variety, cognitive novelty, and changes in cognitive framing (Schoevers, Kroesbergen, Kattou 2020; Zhang, Gan, Wang 2017). Mathematical creativity also promotes self-efficacy (Bicer, Lee, Perihan, Capraro, Capraro 2020; Regier, Savic 2020).

Bicknell and Holton (Bicknell, Holton 2009) argued that mathematical giftedness can be manifested in three ways. The first is the analytic mode – mathematically gifted students figure out problems by using logic and thought. The second is the geometric mode – students will prefer to use sketches and visual aids to figure out problems. The third is the harmonic mode, which represents the gifted students who are capable of both the analytic and the geometric modes.

When it comes to the mathematics teacher who teaches students gifted in mathematics, they should "have access to professional development research information and resources to deal with such issues as identification or recognition of students with mathematical promise, high levels of expectations for all students along with challenging top students to even higher levels of success, pedagogical and questioning techniques to extend students' thinking, and selection and/or development of appropriate curriculum and assessment tools that provide opportunities for students to create problems, generalize patterns, and connect various aspects of mathematics, development of teachers' own mathematical power to make connections and the mathematical sophistication to see the big picture, making appropriate instructional decisions for these promising students, and awareness of, access to and ability to use technology and other tools" (Singer, Sheffield, Leikin 2017: 29). In addition, teachers should continue to strengthen their own mathematical content knowledge and demonstrate the joy of being a lifelong learner of mathematics (Sheffield, Bennett, Berriozabal, DeArmond, Wertheimer 1999). Hoth (2017) suggests that the main element in fostering mathematically gifted students is giving them different learning opportunities. One way to do that could be by creating mind maps during their mathematics classes by students gifted in mathematics.

CHARACTERISTICS OF THE TOPIC OF ALGEBRAIC STRUCTURES

Here are the few definitions for the algebraic structures, from simpler to more complex structures, that students should adopt in the third year of high-school education in the program of the subject Linear Algebra and Analytic Geometry, for the students gifted in mathematics:

Ordered pair $(G,*)$, *where* G *is nonempty set closed under binary operation* $*$ *is groupoid.*

A semigroup $(G,*)$ *is groupoid where the binary operation* $*$ *is associative.*

A monoid is a semigroup with an identity (neutral) element.

A group is a monoid such that each $a \in G$ *has an inverse* $a^{-1} \in G$.

Group G *is Abelian or commutative if* $a*b=b*a$ *for all* $a,b \in G$ *(if binary operation* $*$ *is commutative).*

After Abelian groups, students should learn the algebraic structures with two binary operations: ring, ring with neutral element, and field. Also, students should adopt structure – preserving maps: homomorphism (mapping between two groupoids $(G,*)$ and $(H,\cdot)$ where $f:G\to H, (\forall x,y\in G)\, f(x*y)=f(x)\cdot f(y)$), endomorphism (homomorphism which maps G to G), monomorphism (homomorphism which is also injection), epimorphism (homomorphism which is also surjection), isomorphism (homomorphism which is also injection and surjection), and automorphism (homomorphism which is also injection, surjection and which maps G to G),

There is one common feature about the way that algebraic structures and structure-preserving maps are defined. That feature reflects that more complex structures and mappings are defined through introducing the new property to an already defined mathematical concept (algebraic structure or structure-preserving map). These definitions could be considered as analytic definitions. Under this type of definition, we consider the definitions of the nearest genus concepts and their differences. Aristotle described them as: *Genus proximum et differentia specifica.* For instance, in the definition: *A monoid is a semigroup with an identity element*, semigroup is the nearest genus concept to monoid, and existence of the identity element differentiates the concept of a monoid from a semigroup. With many analytic definitions, as the number of genus concepts and differences is increasing, it gets more and more complex for the students to memorize and adopt all these concepts and connect them in a proper mental scheme.

The importance of having sufficient theoretical knowledge regarding the theme of algebraic structures is high because in most of the concrete problems in this theme, students need to examine the type of given algebraic structure with one binary operation, algebraic structure with two binary operations, and the type of structure-preserving maps. For solving these kinds of problems, students must understand what properties they should examine to determine the type of the given algebraic structure (or mapping), and it is quite hard to memorize properties for all these mathematical concepts individually. So, the best approach for learning these mathematical concepts is to know the relationships between these concepts. This puts the teacher in a position in which he must design and conduct well-structured systematization mathematics classes.

RESEARCH METHODOLOGY

As stated earlier, for the students to adopt the appropriate algebraic structures with understanding, to understand the relations between them as much as possible, and to later apply the theoretical knowledge to concrete problems, it is important that the students systematize the necessary theoretical knowledge. Having in mind that the concepts of algebraic structures are quite abstract and that students don't have previous experience with these concepts, it is important that teacher organize well-structured systematization classes.

For this purpose, it is planned that students systematize appropriate theoretical knowledge by creating mind maps, since mind maps have many of the aforementioned positive aspects. The goal of this research is to determine the effects of creating mind maps by students gifted in mathematics in order to improve their achievements, i.e. to improve their necessary theoretical knowledge.

PROCEDURE AND INSTRUMENTS

In the class which preceded the experimental class, the students were asked to bring thicker and larger paper, as well as crayons and pens in various colors. When asked by the teacher if they had created mind maps during their education so far, 7 or 8 students stated that they had in different subjects during their education in elementary school (geography, biology, and Serbian language), and 3 students made mind maps in math classes in elementary school, but during their high school education they did not create a mind map in any mathematics (mathematical analysis, algebra, or geometry) class.

In the introductory part of the systematization classes, the teacher told the students that their task in the given classes was to create a mind map on which they were to show algebraic structures with one and two binary operations, as well as to create a mind map on which they were to illustrate and connect contents related to mappings. Then the teacher explained to them how the given contents should be connected, with the suggestion that they first prepare a working version of the mind map on a smaller piece of paper, and then, when they create a picture in their minds of how it should look according to their understanding, to translate it into the larger, final form of the mind map. In the first part of the classes, students had the task of presenting the algebraic structures with one and with two binary operations, and in second part of the class, students had the task of presenting mappings (homomorphisms) together with special cases of homomorphisms (endomorphism, epimorphism, monomorphism, isomorphism, automorphism) through a mind map.

During the process of creating mind maps, students flipped through notebooks and textbooks, thus determining the acquired knowledge and then systematically presenting them on paper, in the form of mind maps. It should be emphasized that students created mind maps by working in pairs. The students chose who they would collaborate with, with the aim that during the work the students openly talk, discuss, exchange their opinions, and choose the best ways to present appropriate teaching and learning content. In this way, in addition to developing specific subject competencies, students also developed cross-curricular competencies: lifelong learning, communication, and cooperation.

In order to examine the effects of the given methodological approach, students were tested before and after the systematization classes. Namely, in the final part of the class, which preceded the class of systematization, the students solved Test 1 (see Appendix), in which some correct statements were formulated, as well as statements that were essentially incorrect but were formulated similarly to the correct statements. In the given statements, the students were required to show that they recalled and understood the relations between different but related algebraic structures such as groupoid and semigroup, semigroup and monoid, monoid and group, and then mappings such as homomorphism and monomorphism, monomorphism and isomorphism, epimorphism and automorphism, etc. Therefore, the students were not required to simply reproduce the formulations of the definitions of algebraic structures by stating all the conditions that must be satisfied by the given operation(s) defined on the given set, i.e., all properties of the mapping, but to recognize (based on their knowledge of their properties) which structures are special cases of other structures, i.e., under which new conditions a given structure becomes a structure that represent another, higher level class of structures. Students completed the test by marking the correct statements (by circling the letter in front of the correct statements), while the incorrect statements were to remain unmarked. This was followed by a systematization class where students created mind maps and thus connected their knowledge in suitable schemes. Immediately at the beginning of the class that followed the systematization class, the students were given Test 2, designed in accordance with Test 1, where again some statements were correct, and some were not.

PARTICIPANTS

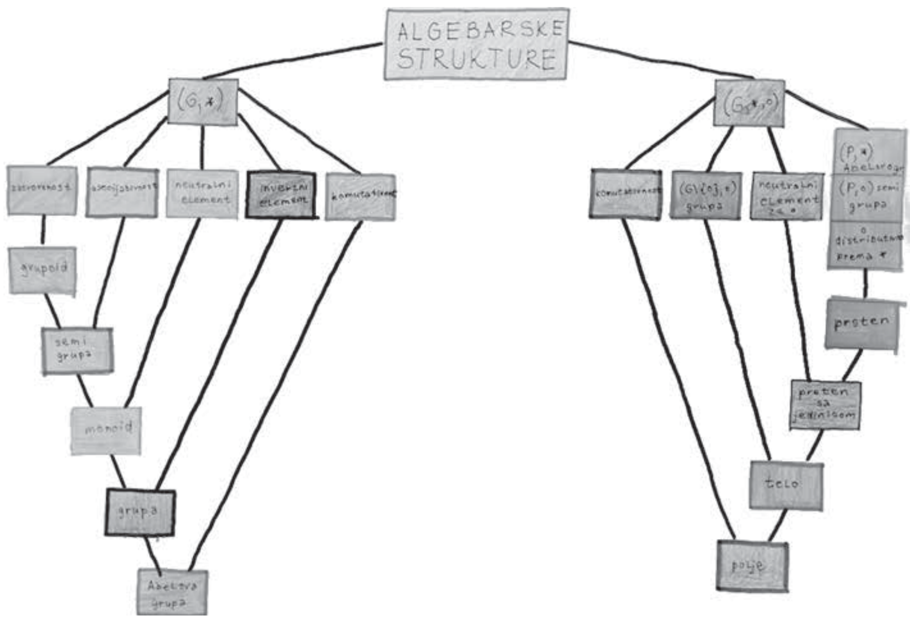
Participants in this quasi-experimental study were 17 third year students gifted in mathematics from the First Gymnasium in Kragujevac who participated in the subject Linear Algebra and Analytic Geometry in the 2021/2022 school year.

RESULTS AND DISCUSSION

ANALYSIS OF THE STUDENTS' WORK IN THE CLASSES

From the students' work, four mind maps are chosen to represent the quality and the mutual characteristics of the mind maps created by students.

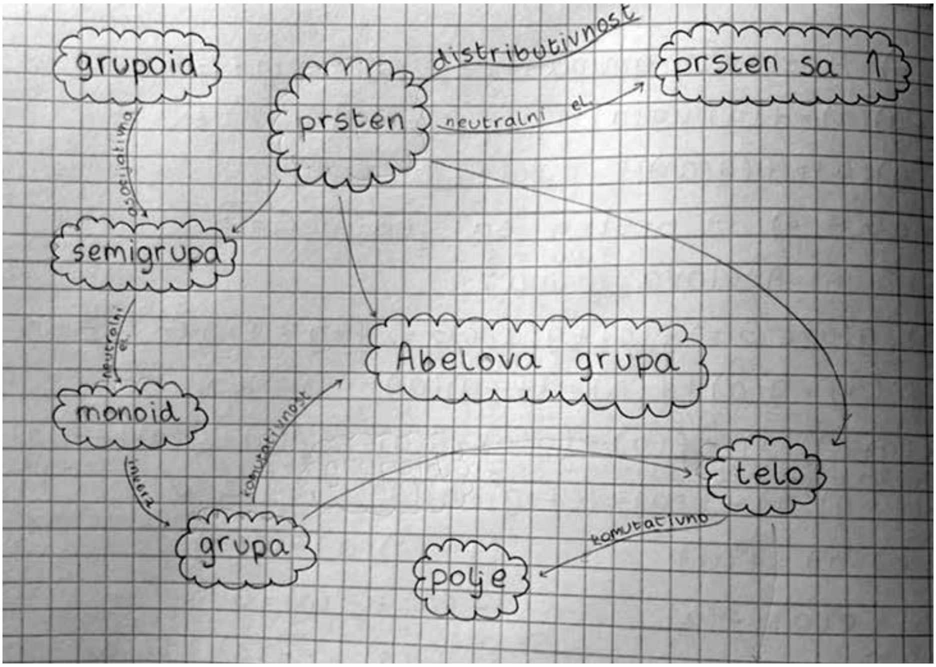
Figure 1. Pair 1 mind map for the theme Algebraic structures



The image presented in the Figure 1 shows a mind map on which the students in the central part, in accordance with the instructions, presented a key term (algebraic structures), then divided the given mathematical concepts into two parts: structures with one binary operation on the left side of the mind map and structures with two binary operations on the right side of the mind map. The students also chose to present the concepts in order of the complexity of the algebraic structures, that is, the number of conditions that the structure must fulfill (the complexity of the structures increases when moving from top to bottom).

The following image presented in the Figure 2 shows a mind map that does not follow the pre-agreed upon structure which a mind map should have. Namely, we can see that to a certain extent the concepts are linearly represented (from groupoid to group), so that the Abelian group is represented in the central part of the mind map, while structures with two binary operations are shown both above (ring, ring with neutral element) and below the centrally represented term (field).

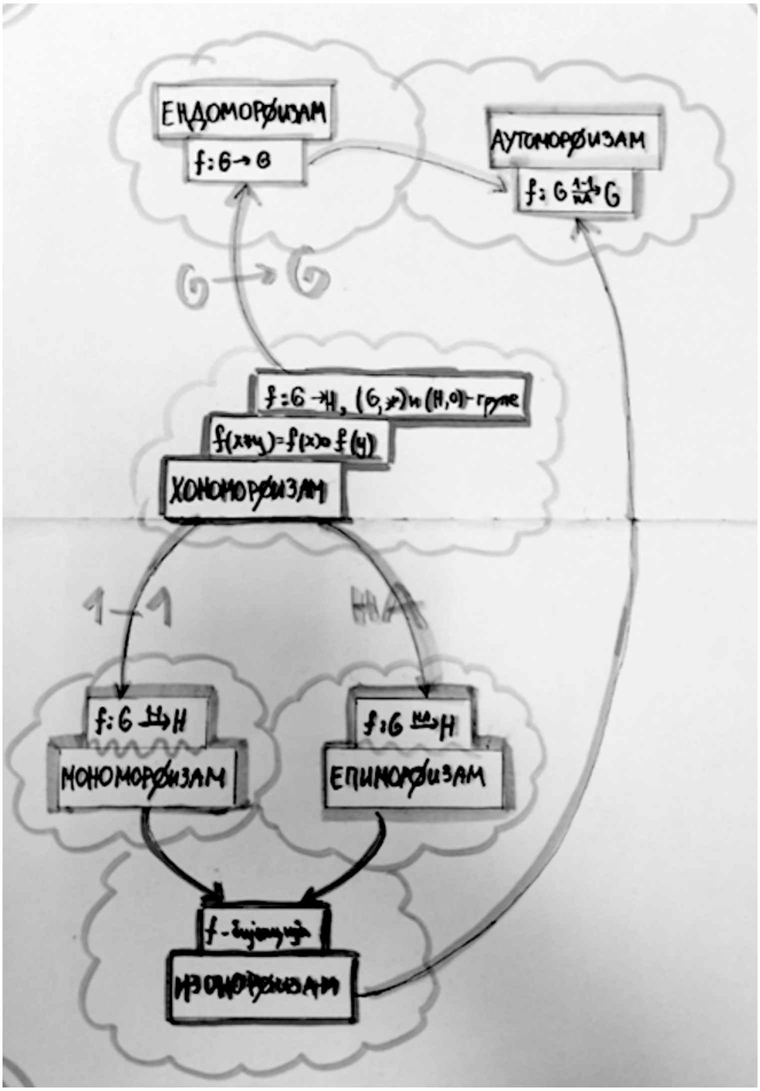
Figure 2. Pair 2 mind map for the theme Algebraic structures



In contrast to the first mind map (shown in Figure 1), the second mind map (shown in Figure 2) is presented more confusedly. It does not have an ideal structure, but still, from the perspective in which the mathematical concepts are connected, it can be concluded that the students master the given concepts and understand their properties and the connections between them.

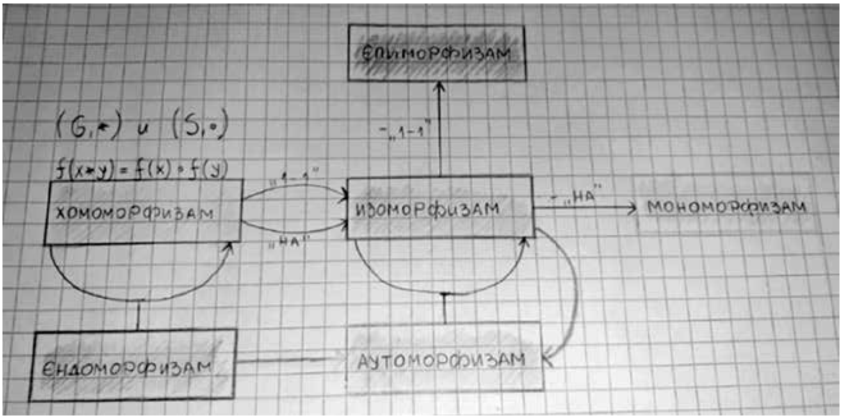
In the Figure 3, we can see a very nicely structured mind map, on which the basic concept (homomorphism) is presented in the central part, together with the definition written in mathematical notation. Furthermore, it can be observed that concepts further branch according to special classes of homomorphisms with precisely written conditions in mathematical notation as well.

Figure 3. Pair 6 mind map for the theme Homomorphisms



The image presented in Figure 4 shows a not quite satisfactorily structured mind map. Namely, the concept of homomorphism is not presented in the central part, but the concept of isomorphism (which represents a homomorphism that is also a bijection). This caused the terms epimorphism (which represents a homomorphism that is a surjection, not an injection) and monomorphism (which represents a homomorphism that is an injection, not a surjection) to be presented as an isomorphism in which one of the properties does not apply (with a “minus” sign).

Figure 4. Pair 3 mind map for the theme Homomorphisms



The first impression based on the analysis of students' work on creating mind maps in the systematization classes is that some students (probably due to a lack of experience in systematically presenting teaching and learning content through mind maps) did not follow the technical instructions for making mind maps, specifically about the way in which concepts should be arranged. The use of colors is also not quite satisfactory. On the other hand (which is extremely important, and which speaks in favor of the fact that the students showed an enviable level of knowledge), there were no material errors in students' work. There were no errors of a mathematical nature on any mind map. In all mind maps, the conditions that certain mathematical concepts must meet were accurately and precisely represented.

Students who took part in the quasi-experimental study were very enthusiastic while creating mind maps and very dedicated to their work. These students' impressions are in accordance with the conclusion of other research (Budd 2004) about students' recognition of the positive impact of learning based on mind maps.

ANALYSIS OF THE STUDENTS' TESTS RESULTS

As said earlier, both tests consisted of a total of 16 statements (see Appendices).

Table 1. Students' results on the Test 1 and the Test 2

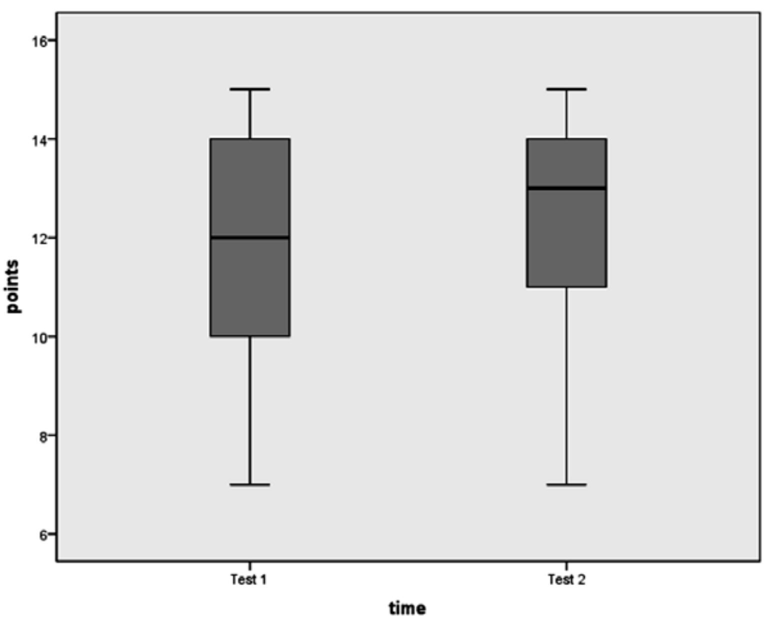
Student	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Before	14	14	7	12	10	12	7	7	12	10	9	15	14	12	14	11	11
After	15	14	11	11	11	14	7	13	14	11	12	13	14	14	14	12	13
Difference	1	0	4	-1	1	2	0	6	2	1	3	-2	0	2	0	1	2

The results of the students' work on these tests are presented in Table 1. As can be seen, for each student who attended all four classes (systematization classes, and classes before and after the systematization classes) the differences (number of points that students achieved after creating mind map minus the number of points students achieved before creating mind maps) in the number of points scored by the students were calculated. Out of a total of 17 students, 12 students achieved a higher number of points on Test 2 compared to the number of points on Test 1. Of the remaining 5 students, 3 students achieved identical results, while two students had more incorrect answers after the systematization classes.

It is interesting that the students who showed the greatest progress in their results (i.e., their knowledge of the given concepts) are those students who achieved lower results and showed a lower degree of interest for the given teaching contents in the third grade within this subject.

Based on the graphic presented in Figure 5, it can be seen from the distribution of the number of points (that students achieved in Test 1 and Test 2), that in most cases, students achieved between 10 and 14 points on Test 1 and between 11 and 14 points on Test 2. Minimums and maximums of points that students achieved are the same for both the tests. Also, the median number of points that students achieved on Test 2 is higher than on Test 1.

Figure 5. Distribution of the number of points that students achieved while solving Test 1 and Test 2



Arithmetic means of the number of points scored by students before (Test 1) and after the systematization classes (Test 2) were calculated. The average number of points achieved by the students before the systematization classes is equal to 11.24, while the average number of points achieved by the students after the systematization classes is equal to 12.54. So, on average, students improved their scores by 1.3 points. Bearing in mind that they could achieve a maximum of 16 points, we notice that the students generally showed an enviable level of knowledge both before and after the systematization classes. This speaks in favor of the fact that during the classes of adopting new teaching and learning content and exercise classes as well, the students adopted and understood the teaching content to a large extent, while after the classes of systematization, which was conducted through the creation of mind maps by the students, they additionally established appropriate connections and relations between different mathematical concepts. This result is in line with other results regarding the potential of using mind maps in order for students to deepen their knowledge (Brinkmann 2003; Kovačević, Segedinac 2007; Papić, Aleksić, Kuzmanović, Papić 2015).

Table 2. Statistical analysis of the students' results

Time	Number of students	Means	Medians	Mean rank	Sum of ranks	Wilcoxon Signed Ranks Test	
						Z	p (2-tailed)
Before	17	11.24	12.00	5.50	11.00	- 2.441	0.015
After	17	12.54	13.00	7.27	80.00		

Based on the results of the non-parametric Wilcoxon rank test, it was found that the number of points that students achieved on Test 2, i.e., on the test which followed the systematization classes, were statistically significantly better compared to the number of points the students achieved on the test held before the systematization classes (Test 1). As earlier confirmed in other empirical research conducted on heterogeneous classes of students (Kovačević, Segedinac 2007), this result speaks in favor of the fact that the students gifted in mathematics significantly systematized and deepened their theoretical knowledge (about algebraic structures and homomorphism) by creating mind maps and systematizing the teaching and learning content, i.e. mind maps contribute to better student achievement when their creation (by students) is implemented in homogeneous classes of students (classes formed with students gifted in mathematics).

CONCLUSION

It is generally known that the teaching contents provided by the Serbian curriculum for secondary school (high school) students are significantly more abstract compared to elementary school, while the teaching methodology is also significantly more formalized. Mathematics teachers, withdrawn from the teaching content, are mostly implementing the frontal form of teaching mathematics, which is even more pronounced in classes with students gifted in mathematics. Examples of some more innovative approaches (not related to solving tasks), except perhaps the occasional use of ICT in teaching with gifted students for mathematics, are very difficult to find, at least in the relevant literature. On the other hand, mind maps have proven to be effective in the implementation of mathematics classes with the aim of students acquiring and understanding the necessary knowledge and connecting mathematical concepts in an appropriate scheme. All these reasons can be considered as the background for the highly motivated students who participated in this quasi-experimental study that aimed to examine whether the creation of mind maps by gifted students in mathematics leads to better student achievement. Indeed, all students participated in the work during the classes and showed an enviable level of commitment.

Based on the analysis of the students' work, it can be concluded that a certain number of students, probably due to the lack of experience in the creation of mind maps, bypassed some agreed technical characteristics that a mind map should fulfill. On the other hand, all mind maps were mathematically correct, with appropriate mathematical notation and without material errors. The results of the tests that the students took before and after the systematization classes indicate that the creation of a mind map with the aim of systematically presenting the teaching content from the Algebraic Structure topic leads to an improvement in student mathematical achievement.

Bearing in mind the small sample size of this research, as well as the fact that the students only systematized the teaching content from one topic in this way, no generalized conclusions can be made, but the results of this quasi-experimental study certainly speak in favor of the implementation of systematization classes in mathematics couses (in analysis, algebra, geometry courses) with students gifted in mathematics. Some future research could follow in order to design and implement several systematization classes during one school year with the students gifted in mathematics (on one or even on two mathematical courses with the same group of students) and additionally to examine the effects of this methodological approach.

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APPENDICES

TEST 1

Mark the correct claims.

a) If algebraic structure $(G,*)$ is groupoid, then $(G,*)$ is semigroup.
b) If algebraic structure $(G,*)$ is semigroup, then $(G,*)$ is groupoid.
c) If algebraic structure $(G,*)$ is semigroup, then $(G,*)$ is monoid.
d) If algebraic structure $(G,*)$ is monoid, then $(G,*)$ semigroup.
e) If algebraic structure $(G,*)$ is monoid in which every element has its inverse element, then $(G,*)$ is Abelian group.
f) If algebraic structure $(G,*)$ is group and if the binary operation $*$ is associative on the set G, then $(G,*)$ is Abelian group.
g) If algebraic structure $(G,+,\cdot)$ is ring and if $(G\setminus\{0\})$ is group, then $(G,+,\cdot)$ is field.
h) If algebraic structure $(G,+,\cdot)$ is ring and if $(G\setminus\{0\})$ is Abelian group, then $(G,+,\cdot)$ is field.
i) If algebraic structure $(G,+)$ is Abelian group, if $(G\setminus\{0\})$ is Abelian group and if multiplication is distributive over addition, then $(G,+,\cdot)$ is field.

j) If algebraic structure $(G,+,\cdot)$ is field, then $(G,+,\cdot)$ is ring.
k) If algebraic structure $(G,+,\cdot)$ is ring, then $(G,+,\cdot)$ is field.
l) If structure-preserving map is monomorphism, then it is homomorphism.
m) If structure-preserving map is isomorphism, then it is monomorphism.
n) If structure-preserving map is endomorphism, then it is automorphism.
o) If structure-preserving map is isomorphism, then it is epimorphism.
p) If structure-preserving map is isomorphism, then it is automorphism.

TEST 2

Mark the correct claims.
a) If algebraic structure $(G,*)$ is groupoid, then $(G,*)$ is monoid.
b) If algebraic structure $(G,*)$ is monoid, then $(G,*)$ is groupoid.
c) If algebraic structure $(G,*)$ is semigroup, then $(G,*)$ is monoid.
d) If algebraic structure $(G,*)$ is monoid, then $(G,*)$ semigroup.
e) If algebraic structure $(G,*)$ is semigroup in which every element has its inverse element, then $(G,*)$ is group.
f) If algebraic structure $(G,*)$ is group and if the binary operation $*$ is commutative on the set G, then $(G,*)$ is Abelian group.
g) If algebraic structure $(G,+)$ is Abelian group, if $(G\setminus\{0\})$ is group and if multiplication is distributive over addition, then $(G,+,\cdot)$ is field.
h) If algebraic structure $(G,+,\cdot)$ is field, then $(G,+,\cdot)$ is ring.
i) If algebraic structure $(G,+,\cdot)$ is ring, then $(G,+,\cdot)$ is field.
j) If algebraic structure $(G,+,\cdot)$ is ring with neutral element, then $(G,+,\cdot)$ is field.
k) If algebraic structure $(G,+,\cdot)$ is field, then $(G,+,\cdot)$ is ring with neutral element.
l) If structure-preserving map is monomorphism, then it is epimorphism.
m) If structure-preserving map is isomorphism, then it is monomorphism.
n) If structure-preserving map is endomorphism, then it is epimorphism.
o) If structure-preserving map is isomorphism, then it is automorphism.
p) If structure-preserving map is automorphism, then it is monomorphism.

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АЛГЕБАРСКЕ СТРУКТУРЕ И ИЗРАДА МАПА УМА ОД СТРАНЕ УЧЕНИКА СА ПОСЕБНИМ СПОСОБНОСТИМА ЗА МАТЕМАТИКУ

Резиме: Рад са ученицима надареним за математику је предмет великог броја студија. Такође, у литератури се могу наћи примери позитивног утицаја употребе мапа ума на учење са разумевањем, повезивањем појмова у одговарајуће схеме, али утицај креирања мапа ума од стране ученика са посебним способностима за математику на њихова постигнућа није довољно истражен. Имајући то у виду, као и да алгебарске структуре представљају наставну тему у којој је неопходно да ученици усвоје одговарајућа теоријска знања о поменутим алгебарским структурама и односима између њих, примењен је овај методски приступ како би ученици дате појмове повезали у одговарајућу шему. У том циљу спроведена су два часа (двочас) систематизације за наставну тему Алгебарске структуре тако што су ученици креирали по две мапе ума (једну за алгебарске структуре са једном и са две бинарне операције и другу за хомоморфизме). Ефекти овог приступа на часовима систематизације су испитивани анализом успеха ученика који су они остварили приликом израде два петнаестоминутна теста (пре и после часова систематизације) на којима је требало да означе тачне тврдње (прецизно формулисане алгебарске структуре и хомоморфизме). Резултати добијени статистичком анализом указују да су ученици постигли статистички значајно боље резултате на тесту одржаном након часова систематизације (у односу на резултате постигнуте на тесту одржаном пре часова систематизације). Другим речима, креирање мапа ума од стране ученика позитивно је утицало на систематизацију знања о алгебарским структурама и на постигнућа ученика (са посебним способностима за математику) из математике (конкретно линеарне алгебре и аналитичке геометрије). Овај резултат имплицира да наставници који раде са ученицима са посебним способностима за математику треба озбиљно да размисле о организовању часова математике на којима ће ученици систематизовати и продубити своја теоријска знања креирањем мапа ума.

Кључне речи: мапе ума, ученици са посебним способностима за математику, алгебарске структуре, настава математике.