TANGRAM

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In this paper we present a brief history of tangram, a Chinese mathematical puzzle/game, which is more than three milleniums old. During the 19th century more than 6500 various problems (figures) related to tangram were solved, and this number is constantly growing. However, it is known that only thirteen convex shapes can be made.

Keywords: Tangram, Chinese math puzzle/game, solution (figure), convex figure.

ABOUT THE TANGRAM

Tangram is one of the oldest and most famous dissection puzzles (Lepšić & Ilić-Dreven, 1981).It is a mathematical puzzle[1], an interesting game which "engages" the brains and contributes to the development of creativity. According to a Chinese psychologist, although it was created for entertainment rather than for analysis, tangram is the oldest psychological test.

The very name *t a n g r a m* is a compound of two words: *tang* and *ram* (Loyd, 1903) and tangram in Chinese means "*seven tiles of wisdom* ".There is little known about the origins of Tangram, which means that the exact age of the game is unknown, but it is considered to be more than three thousand years old.

The legend says that a servant of a Chinese emperor, carrying a very valuable ceramic tile of square shape, tossed and fell and the tile broke into seven parts. Trying to put them in a square shape, the servant created various shapes of animals, people, and things.

1 A *mathematical puzzle* (ora "*brain teaser*") is *a* puzzle based on mathematical rules and the mathematical rules are mostly from the field of numerical theory or geometry. They are intended primarily for one player, who, based on the given conditions, should solve the task, i.e. the puzzle. Most mathematical puzzles cannot be solved without the knowledge of mathematics, as opposed to *mathematical games* where it is not necessary.

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Tangram game arrived in Europe (and other parts of the world) from China in the early 19th century.

T*angram* game gained great popularity in the first half of the 19th century, whereas the second wave of popularity occurred during the First World War.

Among the great tangram lovers were the famous American writer of crime novels Edgar Alan Po, and Napoleon, the French general who spent his days in captivity on the island of Saint Helena playing this game.

This mathematical puzzle consists of seven standard parts (Figure 1) of which the shapes of different objects are made. Traditionally, tangram tiles were made of stone, bone, clay, porcelain and similar materials, but today they are made of plastic, wood or hardboard.

The original tangram is square in shape, the side is one unit[2] (*jd*) long, cut into seven parts (see Figure 1). This way the following parts are obtained:

- two large isosceles right triangles with legs of length $\frac{\sqrt{2}}{2}$ (*jd*) and the hypotenuse of 1 (*jd*), and the area of $\frac{1}{4}$ (jd^2) – in image1 marked with *a*;
- one medium-sized isosceles right triangle with legs of length $\frac{1}{2}$ (*jd*) and the hypotenuse of $\frac{\sqrt{2}}{2}$ (*jd*) and the area of $\frac{1}{8}$ (jd^2) – in image 1 marked with *b*;
- two small isosceles right triangles with legs of length $\frac{\sqrt{2}}{2}$ (*jd*) and the hypotenuse of $\frac{1}{2}$ (*jd*), and the area of $\frac{1}{4}$ (jd^2) – in image 1 marked with *c*;
- one square with sides of length $\frac{\sqrt{2}}{4}$ (*jd*) and the area of $\frac{1}{8}$ (jd^2) – in image 1 marked with *d*; and
- one parallelogram with sides of length $\frac{\sqrt{2}}{4}$ (*jd*) and $\frac{1}{2}$ (*cm*), and the area of $\frac{1}{8}$ (jd^2) – in image 1 marked with *e*.

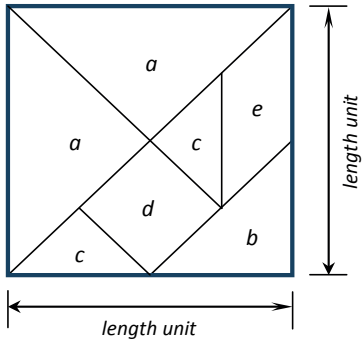

Figure 1.

2 Length units (*jd*) are: 1 *mm*, 1 *cm*, 1 *dm*, etc.

The game involves making predefined, different shapes from the cut pieces, according to one's own idea or, if there are multiple players participating in the game, according to the proposal of the game leader. In doing so, the following basic rules must be obeyed and respected:

1. *All the seven parts must always be used.*
2. *The parts are placed side by side and must not overlap.*
3. *Parts can be turned to the other side when needed.*

For those who have never tried to solve such mathematical puzzles/games this is an opportunity to do it now. And if you have solved such puzzles before, solve them again and return, at least briefly, to the carefree childhood.

In the following example we only give you some ideas and it is up to you to try to think of something new and creative.

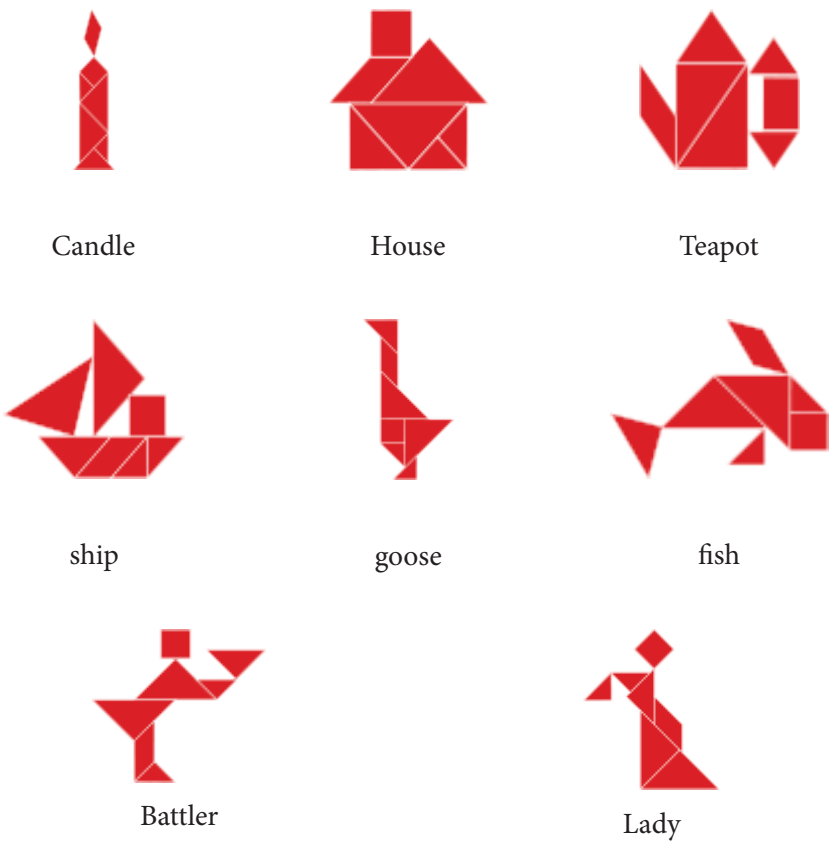

Figure 2. *E x a m p l e s* (taken from: https://hr.wikipwedia.org/wiki/Datoteka:Tangram_203_Nevit.svg)

Solving the tangram requires full concentration and patience and is an excellent exercise for improving the ability to perceive and learn geometric shapes. It is recommended as an additional tool in the education of the youngest.

Only in the 19th century over 6500 different problems were solved and this number is constantly growing. However, only thirteen (13) convex shapes[3] can be made. The thirteen possibilities are shown in Figure 3.

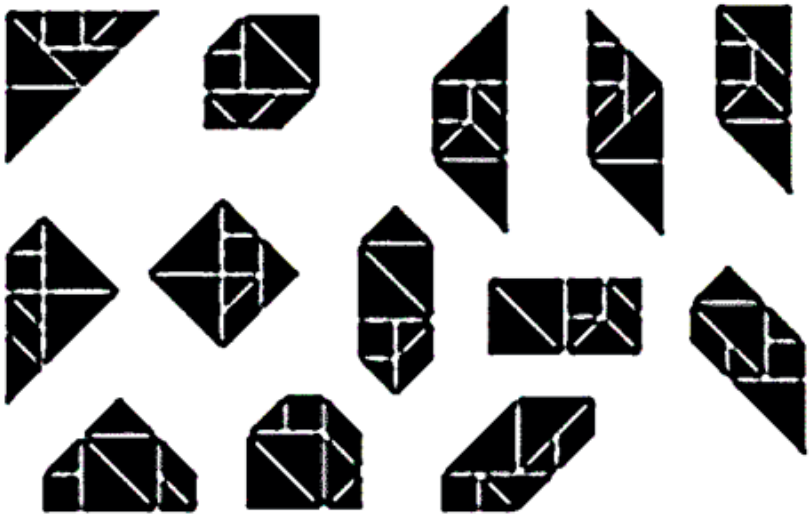

Figure 3.

The famous creators/inventors of such games were Henry Dudeney and Sam Loyd. Martin Gardner is also known for many new games he created / invented and released. Most of such assignments are published in the form of an interesting story or anecdote to avoid the dullness of the mathematical terminology.

A large number of puzzle newspapers and magazines, besides crossword puzzles, regularly publish mathematical puzzles for their readers' entertainment. Numerous mathematical puzzles can also be found and interactively played on the Internet.

THE USE OF THE TANGRAM IN TEACHING MATHEMATICS

In teaching mathematics, we should use games and mathematical puzzles as much as possible, because through mathematical games students develop skills,

3 A convex shape is a shape on which the length drawn between any two points on the perimeter lies completely inside the shape.

creativity, imagination and interest. They can be played for fun, but also for acquiring knowledge; they last relatively a short time and do not require special equipment; they are suitable for all ages and good knowledge of school mathematics is not necessary (Borković, 2017).

We can start playing the tangram game with students of the youngest age, i.e. with first and second graders. We may ask students to distinguish geometric shapes and try to design certain shapes by using only the shapes of one group, for example, all triangles.

During mathematics classes, third and fourth graders of elementary school can be given the assignment of drawing a tangram and cutting out its components. Then, they can continue with the stacking game where they can only use some parts and later be allowed to use all the remaining parts.

In higher grades of elementary school, students can be asked to determine the area of each individual shape which is part of the tangram if the area of the smallest shape is defined, i.e. if its area is *P*. Finally, based on the calculated areas of all individual shapes, they should be asked to define the area of the square as a function of the area *P*.

Student teachers doing the courses *Methodology of teaching mathematics* and *Teaching gifted students* can be given the assignment to create their own tangram by respecting the rules of making a tangram. Students can be asked to determine the area of each individual shape and the whole tangram as a function of the P area of the smallest shape. Further, students can be asked to determine the perimeter of each individual shape. In the end, they can try to create certain shapes or to create as many shapes as possible.

Here are some of the tasks that can be given to school or university students.

1. How many shapes are there in a tangram? (*seven*)
2. Which geometric shapes does the tangram consist of ? (*five isosceles right triangles, one square and one parallelogram*)
3. Are there any congruent, "equal" shapes among them? (*the two large triangles are congruent; the two small triangles are congruent as well; the remaining shapes are not congruent)*
4. If the area of the smallest triangles is *P*, compare the areas of all parts of the tangram. (*the medium-sized triangle is twice the size of the smallest triangle; therefore, its area is 2P; the largest triangle has the area twice larger than the medium-sized triangle; therefore, its area is 4P; the area of the square is twice larger than the area of the smallest triangles and its area is 2P; the area of parallelogram is twice larger than the area of the smallest triangle; therefore, its area is 2P* – methodical remark: everything mentioned should be clearly demonstrated by "covering" the larger shapes with smaller ones).

5. What is the area of the tangram? (*Now that we know the areas of all the parts of the tangram – see the previous task – the total area of the tangram is 16P which means that the area of the tangram is 16 times larger than the area of the smallest triangle.*)
6. Using two small triangles and a triangle of medium size, compose the following geometric shapes: (1) square; (2) rectangle; (3) triangle; (4) parallelogram /rhombus/; (5) trapezium.
7. What are the areas of such compound shapes /previous task /? (*all the shapes consist of the same parts, so it is easy to conclude that their areas are the same; this means that the area of all these shapes is the same and it is 4P; it can be concluded that d i f f e r e n t geometric shapes can have the s a m e areas.*)
8. Using the two small triangles, the medium-sized triangle, the square and the parallelogram, compose the following geometric shapes: (1) square; (2) rectangle; (3) triangle; (4) parallelogram; (5) trapezium.
9. Using all the seven parts of the tangram, compose the following geometric shapes: (1) square; (2) rectangle; (3) triangle; (4) parallelogram; (5) trapezium.
10. School (or university) students are already given compound shapes and their task is to find specific geometric shapes and/or some other shapes in them.

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