MATHEMATICS, ITS HISTORY AND TEACHING

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The history of mathematics, as a part of general human culture, has a wide subject of research. It represents a base of modern scientific methodology and is one of the most important sources of a thinking process. It researches the sources and ways of generating mathematical concepts and theories, points to the extent these concepts and theories are connected with practice, describes the development of aspirations for generalizing and proving scientific assumptions, etc. The facts that are studied in teaching mathematics originated many centuries ago, and can, therefore, be understood only in a historical context and period. The paper will consider the concept of mathematics, point to the periodization of its development and connect the history of mathematics with teaching.

Keywords: History of *mathematics, educational role of mathematics, wider introduction of the history of mathematics into teaching.*

WHAT IS MATHEMATICS

The word mathematics originates from the Greek word *mathes is* that means 'learning' or 'knowledge'. Mathematics represents a common language of all sciences, technology and other human activities. Its objects are not real. A point, a straight line, a number, a plane, etc. do not exist in the real world. The same mathematical formula can be often applied to the most varied situations in reality which have the same structural characteristics in a logical and quantitative sense. Thus, for example, linear equity $y = kx$ can reflect:

- Dependence between acceleration and force, at constant mass, i.e. $F = m \bullet a$;
- Dependence between volume and mass, at constant density, i.e. $m = p \bullet v$
- Dependence between time and path, at constant speed, i.e. $s = c \bullet t$,etc.

The issue that interests mathematics in quoted and similar examples is just a relationship between sizes of some phenomena, regardless of specific characteristics of these phenomena.

The subject of studying mathematics has changed from a period to a period. For example, ancient mathematics of Egypt, China, Babylon and India dealt with arithmetic exclusively. Later, mathematics became a science of numbers. Presently, there are hundreds of mathematical disciplines and it is very difficult to determine its subject. The simplest way to define the subject of mathematics is to say that mathematics studies characteristics of different objects (numbers, geometrical figures, sets, equities, etc.) and relationships among them, regardless of the origin of these objects.

PERIODIZATION OF HISTORICAL DEVELOPMENT OF MATHEMATICS

Subject of the history of mathematics is an origin and development of mathematical ideas, methods and algorithms. In order to organize and present the immense knowledge of mathematics it is necessary to determine a periodization type of the development of mathematics, i.e. to determine the phases of its development. Determining periodization criteria represents a great problem. Many mathematicians determine a periodization type according to a chronological criterion. One of the most general types of periodization was determined by Kolmogorov (Kolmogorov, A.N., 1903-1987), a great Russian mathematician. A methodological principle Kolmogorov applied in his periodization type is transition from a lower degree abstraction to a higher degree abstraction. He suggests four periods of the historical development of mathematics:

1. Originating of mathematics
2. Elementary mathematics
3. Originating of mathematics with variable values
4. Modern mathematics

The first period lasted from the very beginnings of human existence till the 6thcentury B.C. In this period mathematics was of practical character exclusively; there was no proving nor generalizing or theoretical foundation.

The second period began in the 6thcentury B.C., when mathematics raised to the level of a science and when its foundations were set. Mathematics was differentiated by its subject of research and its methods. Geometry, elements of the theory of numbers, algebra and trigonometry were formed in this period. This period ended at the beginning of the18thcentury with the appearance of Descartes' coordinate geometry in 1637, when the third period began, in which new mathematical ideas originated.

In the third period there was a turbulent development of the complete human civilization. It was the time of new geographical discoveries, seafaring and

astronomy developed, as well as manufacture artillery, etc. All that demanded new ideas and new mathematical solutions, elementary geometry and algebra being insufficient to fulfill new demands. Firstly, mathematics was then interested in variable values and their interdependence – functions. Analytic geometry and mathematical analysis were established.

The 19th century could be marked as the beginning of the fourth period of the Kolmogorov periodization. This period, above all, has been characterized by wideness of mathematical research subjects. New, non-Euclid's geometries were created. Number was no longer the main object in algebra in operations, but it operated with objects of a general character. Group theory and linear algebra appeared then. The sets theory influenced the general path of mathematics. New branches developed in analysis (for example, the theory of the real variable function), as well as topology, functional analysis, etc. Mathematical logics developed, and completely new fields also appeared, such as: the theory of algorithms, the theory of information, operational research, cybernetics, and discrete and final mathematics were born as well.

THE HISTORY OF MATHEMATICS AND TEACHING MATHEMATICS

- Some of educational goals which are achieved by application of mathematics in teaching are:
- Serious science is presented to children more easily through the history of mathematics. Teaching becomes more interesting and livelier.
- Pupils notice that mathematics is a product of a creative activity of the human genius during thousands of years, and not a momentary inspiration of "some clever people". Each mathematical model is a result of catching impulses of nature through centuries.
- The pupils will learn that they should endure a long and difficult path to doing creative work, and that they should make a lot of mental effort in the process, just as the previous generations used to do.
- When doing creative work, besides a series of routine mathematical actions which can be done by every mathematician, there is often at least a decisive one which is the result of a unique idea. The history of mathematics possesses a series of enlightening examples about this matter. One of the most beautiful proofs is the Euclid's proof that prime numbers are infinite.
- Biographies of great people can often help us make useful conclusions which can influence pupils in an informative and educational way.
- If mathematical theories are regarded through their final formulation only,

without their historical interpretation, pupils could get a wrong notion of mathematics as some artificial creation, which serves to wise, mental imaginations, without any connection with practice and applications in it. The importance of studying mathematics will be clearer to pupils when, through historical facts, they understand that mathematics, since its origin, has had one of the most important roles in all activities of a human being.

- A class of mathematics along with treating historical facts receives a cultural dimension.
- Pupils can easier acquire knowledge if they follow a historical way of developing a concept. The history of mathematics should depict to the pupils the origin of a concept at its source and point to the natural and logical way to its highest abstraction. It would be very useful for the pupils to understand the way the creators used to go along, how the process from the idea to announcing the results has gone.

The teachers can educate themselves through the following resources on the history of mathematics in teaching:

1. *Seminars*: it is suggested that professional associations should organize seminars in which teacher practitioners would be educated in connection with usage of the history of mathematics in teaching.
2. *Magazines*: it is suggested that the themes from the history of mathematics should be included much more in the magazines for pupils. These themes should be in line with the curriculum. The history of mathematics themes should also find more place in magazines intended for the teachers.
3. *The textbook*: it is proposed to write special textbooks on the history of mathematics. This kind of practice is known in Russia, France, Italy and Romania. The books should be adapted to two levels of education, i.e. elementary and secondary school. The contents of textbooks like this should completely follow the curriculum of mathematics. The teacher should also examine all historical facts related to a teaching unit that has been taught as well within the preparation for work.
4. *The lexicon* of mathematical signs and symbols can be very useful, especially at introducing new mathematical symbols. Then, from the lexicon, the historical path should be recognized, the reasons for its specific form, the person that was the first to use this symbol, the qualities of the other forms, etc.

Methodology recommendation is not to exaggerate with historical facts and not to blur the essence of the theme that has been taught. When a name of some mathematician is mentioned, it is sufficient to retell some anecdotes from his life, to enlighten his work or some concept related to his name. To help children master the concept which has been taught, a short historical development of the concept

should be given to them. It is sufficient to spend up to five minutes in each class discussing historical facts; moreover, this will significantly improve the discipline during the class. Besides telling indispensable historical facts related to concepts taught in regular teaching, we can introduce elements of the history of mathematics in additional classes. The pupils should elaborate historical themes for several days, and after that, discuss the elaborated themes. Many of the same historical issues could be taught within the scope of different grades and in various contexts.

CONCLUSION

Some theoretical opinions have been briefly presented in connection with the theme of the history of mathematics and its application in teaching. A general conclusion is that the history of mathematics is presented insufficiently in teaching mathematics and that its presentation depends on the willingness of teachers to use it more widely. However, there are ways for presenting it more in teaching. The most efficient way is to incorporate elements of the history of mathematics into the textbooks of mathematics, in places where it is necessary, and to the right extent. It is necessary to study concrete examples of application of the history of mathematics in teaching mathematics and to give methodology recommendation for their application in teaching. It is also necessary to survey the teachers' attitudes to this theme and in accordance with those attitudes to make a concrete curriculum for wider introduction of the history of mathematics into teaching.

The paper also presents a short list of literature relevant to the theme, which can be used in a longer paper.

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