ABU'L-WAFA PROBLEM – POSSIBLE TOOL FOR FOSTERING SUBJECT AND PEDAGOGICAL CONTENT KNOWLEDGE OF PRE-SERVICE MATHEMATICS TEACHERS

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In this paper we present the problem posed by medieval Middle Eastern mathematician Abu'l-Wafa, as well as different solutions of this problem. Solving Abu'l-Wafa's problem could be a powerful tool for building and fostering subject content knowledge and pedagogical content knowledge of pre-service mathematics teachers. In solving this problem students have to use many geometrical concepts. On the other hand, as future mathematics teachers, they have to find out when and how to engage pupils to solve this problem, considering the question of procedural and conceptual knowledge in mathematics as well as the important question of the role of proof and argumentation in mathematics classes.

Keywords: Abu'l-Wafa problem, mathematical content knowledge, pedagogical content knowledge.

INTRODUCTION

In geometry classes, apart from other problems, we solve construction problems, i.e. in given conditions, we draw a geometrical figure using only a ruler and a compass. Solving construction problems brings several benefits to pupils: they learn and use construction procedures, connect geometrical concepts, provide proofs for proposed procedures, discuss different aspects of problem's conditions and connect them with possible solutions. Furthermore, when solving this type of problems, pupils should precisely and accurately express geometrical ideas in a symbolic, graphical and textual representation.

In next chapter we will present different solutions to a problem posed by Middle Eastern mathematician Abu'l-Wafa. This problem could be posed to pupils of different ages, either in middle or high school.

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ABU'L -WAFA PROBLEM

Abû'l-Wafa Buzjani (940-998) was born in Buzjan, near Nishabur, a city in Khorasan, Iran. He learned mathematics from his uncles and later on, when he was in his twenties, he moved to Baghdad. He flourished there as a mathematician and astronomer. In his treatise titled *On Those Parts of Geometry Needed by Craftsmen,* Abu'l-Wafa described several constructions made with the aid of a ruler and a "rusty compass", a compass with a fixed angle. These included constructing a perpendicular at the endpoint of a line segment, dividing segments into equal parts, bisecting angles, constructing a square in a circle and, constructing a regular pentagon (Berggren, 2003).He was one of a long line of Islamic mathematicians who developed geometric techniques that proved useful to artisans in creating the highly symmetrical ornamentation found in architecture around the world today (Tennant, 2003).

Problem: *Construct at the endpoint A of segment AB a perpendicular to that segment, without prolonging the segment beyond A.*

The problem may be solved in different ways.

Solution 1: Abu'l-Wafa's solution (Berggren, 2003)

Procedure. On AB mark off with the compass segment AC and with the same opening, draw circles centered at A and C, which meet at D. Extend CD beyond D to E so that ED = DC. Then $\measuredangle CAE$ is a right angle (Figure 1).

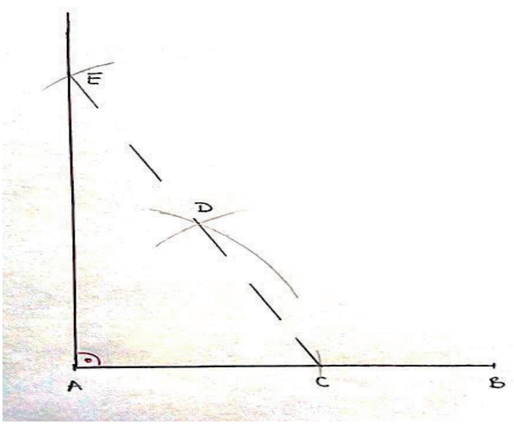

Figure 1. *Solution 1*

Proof. The circle that passes through E, A, C has D as a center since DC = DA = DE. Thus, EC is a diameter of that circle and therefore $\sphericalangle CAE$ is the peripheral angle subtended by the diameter of the circle. Each such angle is the right angle (Tales theorem about angle subtended by diameter of circle).

Remark. In order to perform this solution pupil should use notions of central and peripheral angle as well as Thales theorem about angle subtended by diameter of circle.

Solution 2:

Procedure. Using a compass construct angles: $\sphericalangle pAq$ = 60° and $\sphericalangle qAr$ = 60°. Construct bisectors of $\sphericalangle qAr$. Angle $\sphericalangle pAs$ is the right angle, i.e. s in perpendicular to segment AB (Figure 2).

Proof. Angles $\sphericalangle BAq$ = 60° and $\sphericalangle qAr$ = 60° are adjacent angles. Bisectors of angle $\sphericalangle qAr$ passes through A and divide $\sphericalangle qAr$ into two equal angles $\sphericalangle qAs$ = 30° and $\sphericalangle sAr$= 30°. Angles $\sphericalangle BAq$ and $\sphericalangle q$As are adjacent angles. Their sum is angle $\sphericalangle BAs$.

$\sphericalangle BAs = \sphericalangle BAq + \sphericalangle qAs$ = 60°+ 30° = 90°.

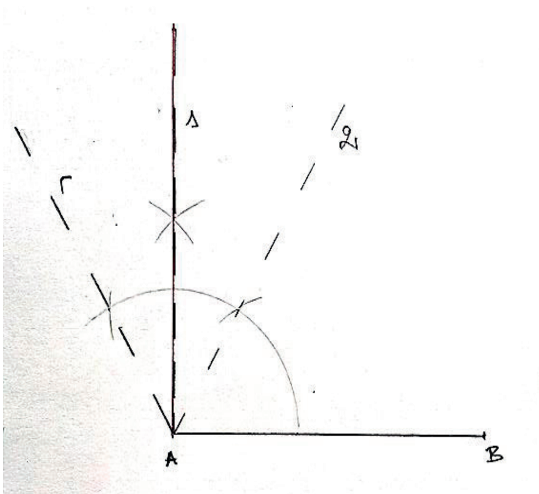

Figure 2. *Solution 2*

Remark. This solution uses construction of angles of 30°, 60° and 90° and is suitable for pupils in early grades of middle school.

Solution 3:

Procedure. Construct the bisector of the segment AB. Translate the bisector to the endpoint A of segment AB (Figure 3).

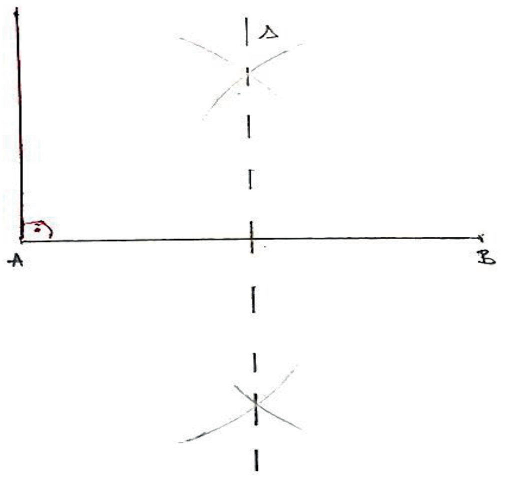

Figure 3. Solution 3

Proof. The bisector of segment AB is perpendicular to AB. Now we use a property of isometry: each translation maps a right line into a parallel right line.

Remark. This solution does not represent geometrical construction because two rulers were used to translate bisector s to the endpoint A. However, this solution is appropriate when pupils learn about isometries in the middle school.

USE OF ABU'L-WAFA PROBLEM TO FOSTER CONTENT AND PEDAGOGICAL KNOWLEDGE OF MATHEMATICS TEACHERS

Mathematics teachers have to master mathematical contents, as well as pedagogical content knowledge that will enable them to create an environment in which pupils will learn in an optimal and effective way. In other words, the teacher must be able to adapt mathematical content to the age of the student, without compromising the principle of science. In mathematics teaching and learning it is equally important to develop both procedural and conceptual knowledge among pupils. Mathematics teachers need to find a "right measure" in teaching, paying attention both to performing mathematical procedures as well as to giving explanations, argumentations and proofs. They have to promote such mathematical culture among their pupils. Using episodes from history of mathematics could help teachers to accomplish this.

Integration of the history of mathematics in teaching and learning mathematics could be justified by several arguments. Tzanakis and Arcavi (2000) point out the following five fields in which integration of the history of mathematics may be particularly relevant to support, enrich and improve the teaching and learning process:

- learning of mathematics;
- nature of mathematics and mathematical activity;
- didactical background of teachers;
- affective predisposition towards mathematics;
- appreciation of mathematics as a cultural endeavor.

CONCLUSION

Taking into account previous considerations, the Abu'l-Wafa's problem can be used to build and foster subject content knowledge and pedagogical content knowledge of pre-service mathematics teachers.

Abu'l-Wafa's problem will take a central place in our case study among pre-service mathematics teachers. The goal of this study is to foster subject content knowledge and pedagogical content knowledge of a group of pre-service mathematics teachers by using the episode from history of mathematics.

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