FANGCHENG METHOD AS A TOOL FOR DEVELOPING PRE-ALGEBRA CONCEPTS IN PRIMARY GRADE STUDENTS

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In this paper we give an overview of the Ancient Chinese fangcheng method for solving systems of linear equations and discuss some possibilities of using this method in teaching primary grade students. The history of mathematics shows us that students very often form mathematical concepts in a way similar to the ways these concepts have been formed through the history of mankind. Taking into account these and the results of previous research, we believe that it is possible to use fangcheng method to develop some pre-algebra concepts in primary grade students.

Keywords: Fangcheng method, primary grade students, systems of linear equations

INTRODUCTION

The idea of integrating contents of the history of mathematics in mathematics teaching and learning is not new. There is extensive literature which offers a variety of reasons why and how to use history of mathematics in mathematics education. Wilson and Chauvot (2000) put forward four main benefits of using the history of mathematics in the classroom: (a) it sharpens problem-solving skills, (b) it lays foundation for better understanding of mathematics, (c) it helps students to make mathematical connections, and (d) it highlights interaction between mathematics and society. Liu (2003) gives five arguments for using the history of mathematics in school mathematics: (a) it helps to increase motivation and develop positive attitude toward learning, (b) the analysis of some past obstacles in the development of mathematics may explain the difficulties students face, (c) historical problems may help to develop students' mathematical thinking, (d) history reveals the humanistic aspect of mathematics knowledge, and (e) it gives teachers a guide for teaching. A number of studies points out that it is very important

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for future mathematics teachers to get to know the genesis of mathematical concepts and statements (Schubring et all., 2000; Dejić & Mihajlović, 2014). Guliker & Blom (2001) state that didactic skills of teachers are to be improved by reading old sources. It helps teachers to understand why certain concepts are difficult for their students. In the process of mathematics teaching and learning, the order of topics in the curriculum is determined mostly in accordance with historical development of some mathematical ideas (Katz, 1993). Furthermore, students very often form certain mathematical concepts in a similar way these concepts have been formed through the history of mankind: direct counting, measuring, observation of the real objects, etc. (Dejić & Mihajlović, 2014). Clearly, students do not go through the complete historical development (which sometimes lasts for centuries) in learning these mathematical concepts, but use shorter routes which can be facilitated by appropriate methodological transformation of mathematical contents.

In this paper we will discuss possibilities to introduce some pre-algebra concepts such as systems of linear equations to primary grade students by using Ancient Chinese fangcheng method. Since, there are not many empirical research studies that explore the benefits of using history of mathematics contents in mathematics classroom (Jankvist, 2009), we believe that our research will give a significant contribution to the field.

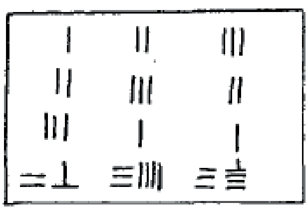
FANGCHENG METHOD

One of the most important mathematical works in China's long history is the *Jiuzhang Suanshu* or *Nine Chapters on the Art and Calculation* (Schwartz, 2008). The authors and date of the paper are not known, but it is believed that it was written shortly after 200 BC. The original version contained rules and algorithms, but no formal proofs or explanations. In the 263 AD Chinese mathematician Liu Hui provided written commentaries and justification for the techniques used. In the eighth chapter of the book, entitled *Fangcheng*, eighteen riddles/problems dealing with systems of linear equations and their solution techniques can be found. The word fangcheng can be translated as *matrices* or *rectangle arrays*. The problems/riddles were set up in a way similar to the way we would today formulate them in Linear algebra using n equations with n unknowns. They were based on practical applications to the real life situations. Chinese computation procedure was performed on a grid called counting board with a set of rods. Various rod configurations represented various numbers. The rod numerals were placed on the counting board in almost the same way we form the matrix for solving equations with multiple unknowns. The directions for solving problems required the setting up and manipulation of rod numeral rectangular arrays. The use of a counting board allowed Chinese to easily distinguish between different variables (Swetz, 1979). The analysis of these manipulations reveals that they used the same principle, which was developed centuries later in 1810 by Carl Friedrich Gauss, known as Gaussian elimination.

We will explain the counting board approach to the first problem given in Chapter 8, which involves a harvest of the three different grades of rice.

Now given 3 bundles of top grade paddy, 2 bundles of medium grade paddy, [and] 1 bundle of low grade paddy. Yield: 39 dou of grain. 2 bundles of top grade paddy, 3 bundles of medium grade paddy, [and] 1 bundle of low grade paddy, yield 34 dou. 1 bundle of top grade paddy, 2 bundles of medium grade paddy, [and] 3 bundles of low grade paddy, yield 26 dou. Tell: how much [dou] does one bundle of each grade yield? (Schwartz, 2008)

The problem can be represented on the counting board (on the left) and by Arabic numerals (on the right) as given in Figure 1.



(I)

Left	Centre	Right
1	2	3
2	3	2
3	1	1
26	34	39

(II)

Figure 1. The counting board and Arabic numerals representation (Martzloff, 2006)

If we used the familiar algebraic notation, then this representation would be equivalent to the following set of equations

$$3x + 2y + z = 39$$
$$2x + 3y + z = 34$$
$$x + 2y + 3z = 26$$

After carrying a series of multiplications and substractions with the appropriate columns, the initial "rectangular array" is reduced to a tabular form as illustrated in Figure 2.

1	6	3
2	9	2
3	3	1
26	102	39

1	0	3
2	5	2
3	1	1
26	24	39

3	0	3
6	5	2
9	1	1
78	24	39

0	0	3
4	5	2
8	1	1
39	24	39

0	0	3
20	5	2
40	1	1
195	24	39

0	0	3
0	5	2
36	1	1
99	24	39

Figure 2. Representation of solution on the counting board

We notice that the matrix of the system is reduced to a triangular form. By using a simple division, we can determine the value of one of the unknowns ($z=36/99$), and the other two unknowns can be found by successive substitutions. Martzloff (2006) points out that the fangcheng technique is "visibly nothing other than Gauss's method" (p. 254), although Gauss's work has little to do with the work of Chinese authors.

POSSIBILITES OF USING FANGCHENG METHOD TO DEVELOP PRE-ALGEBRA CONCEPTS OF PRIMARY GRADE STUDENTS

From the pedagogical point of view, history of mathematics has a potential to be used in school curricula. As Costa et al. (2015) point out, it can contribute to the development of the pre-algebra contents, such as systems of linear equations, unknowns and matrices. The same authors performed a case study about the use of a simplified form of fangcheng method for solving linear system of equations by a talented 10 year old student. Their study showed that it was possible to learn fangcheng method much earlier than the solving systems of linear equations is usually taught.

In Serbian primary schools students in lower grades learn how to solve some simple linear equations with one unknown, and how to model mathematics word problems into these equations. They learn how to solve systems of linear equations in 8^{th} grade. However, in order to solve some complex word problems which require knowledge of some more advanced methods (such as systems of linear equations), primary school students (aged 9 and 10) use some mathematical models which allow them to transform these problems into more simple ones. Some of these models are: a line segment model (which corresponds to a bar model), a rectangle area model, and a false assumption method. For example, the rectangle area model is used in the case when one variable in the mathematics word problem can be expressed as the product of two other variables. The main idea is to present the relation of the three variables through the relation between the sides of the rectangle and its area. If one side of the rectangle is decreased, the other side should be increased in order for area to remain the same. By using a graphical representation in this model, the system of two linear equations with two unknowns is transformed into a process of solving the equation with one unknown.

Fangcheng method also relies on an adequate visual representation of numerals on the counting board and involves only basic calculations, such as multiplication and substraction. If we take into account these and the research by Costa et al. (2015), there is a reason to believe that primary grade students can understand and use the Fangcheng method to solve some mathematics word problems that involve systems of linear equations.

CONCLUSION

The history of mathematics offers teachers a number of guidelines how certain mathematical contents can and should be taught. However, there should be more empirical research in order to explore the effects of using history of mathematics in mathematics teaching. We plan to conduct an experimental study with 4^{th} grade primary students (age 10) and explore the possibilities of using fangcheng method to solve mathematics word problems. The main goal of our study will be to investigate if primary grade students are able to learn and understand the fangcheng method.

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